

A multi-channel fixed point for a Kondo spin coupled to a junction of Luttinger liquids

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Abstract. – We study a system of an impurity spin coupled to a junction of several Tomonaga-Luttinger liquids using a renormalization group scheme. For the decoupled S -matrix at the junction, there is a range of Kondo couplings which flow to a multi-channel fixed point for repulsive inter-electron interactions; this is associated with a characteristic temperature dependence of the spin-flip scatterings. If the junction is governed by the Griffiths S -matrix, the Kondo couplings flow to a strong coupling fixed point where all the wires are decoupled.

Although the Kondo effect has been studied for many years and is one of the best understood paradigms of strongly correlated systems [1], it continues to yield new physics, such as in its recent manifestations in quantum dots [2, 3]. For quantum dots with an odd number of electrons, a Kondo resonance occurs at the Fermi level of the leads, which is seen as a peak in the conductance.

If the leads are one-dimensional, inter-electron interactions turn them into Tomonaga-Luttinger liquids (TLLs); this changes the physics considerably. The Kondo effect has been studied for a system with two TLL leads [4–7] and for crossed TLL wires [8]. For weak potential scattering, the strong coupling fixed point (FP) consists of decoupled TLLs, the conductance vanishes at $T = 0$ via the usual TLL power law [9], and a spin singlet is formed if the impurity has spin $1/2$ (Furusaki-Nagaosa point) [5].

Motivated by recent experiments which probe the Kondo density of states in a three terminal geometry [10], we will study what happens when an impurity spin is coupled to a junction of more than two quantum wires which are modeled as TLLs. The junction is characterized by an S -matrix which governs the connections between the wires, and the couplings of the Kondo spin are described by a J -matrix. (Although the S -matrix formalism for calculating the conductance is strictly valid only for non-interacting electrons, we will use it here because the interactions will be assumed to be weak and will be treated perturbatively [11, 12]).

We obtain the renormalization group (RG) equations for the system, and find that the flow of the Kondo couplings depends on the form of the S -matrix. At the point where all the wires are decoupled from each other, we find that for a large range of initial values of the Kondo couplings, the system flows to a multi-channel FP lying at zero coupling. This FP is

associated with spin-flip scatterings of the electrons from the impurity spin whose temperature dependence will be discussed below. [Note that in the language of the standard N -channel Kondo problem, the scattering matrix S is the $N \times N$ identity matrix, and $J_{ij} = J_i \delta_{ij}$; anisotropy between the channels is introduced by having off-diagonal elements in the S and J matrices]. On the other hand, at the Griffiths S -matrix (defined below), there is no stable FP for finite values of the Kondo couplings, and the system flows towards strong coupling in two possible ways. In one case, the impurity is strongly and antiferromagnetically coupled to the electron spin at the junction. We then perform an expansion in the inverse of the Kondo couplings and find that the system is now near the decoupled S -matrix; hence it flows to the multi-channel FP. In the other case, the impurity is coupled strongly and ferromagnetically to the electron spin at the origin and antiferromagnetically to the neighboring electron spins; further analysis then depends on the values of N and the impurity spin S .

We begin with N semi-infinite wires which meet at a junction, where the incoming and outgoing fields are related by an $N \times N$ unitary S -matrix. For an electron which is incoming in wire i ($= 1, 2, \dots, N$) with spin α ($= \uparrow, \downarrow$) and wave number k (defined with respect to the Fermi wave number k_F), the wave function is given by

$$\begin{aligned} \psi_{i\alpha k}(x) &= e^{-i(k+k_F)x} + S_{ii} e^{i(k+k_F)x} \quad \text{on wire } i, \\ &= S_{ji} e^{i(k+k_F)x} \quad \text{on wire } j \neq i. \end{aligned} \quad (1)$$

Here k goes from $-\Lambda$ to Λ . The second quantized annihilation operator corresponding to the above wave function is given by $\Psi_{i\alpha k}(x) = c_{i\alpha k} \psi_{i\alpha k}(x)$. If an impurity spin is coupled to the electrons at the junction, the Hamiltonian is given by

$$\begin{aligned} H_0 + H_{\text{spin}} &= v_F \sum_i \sum_\alpha \int_{-\Lambda}^{\Lambda} \frac{dk}{2\pi} k c_{i\alpha k}^\dagger c_{i\alpha k} \\ &+ \sum_{i,j} \sum_{\alpha,\beta} \int_{-\Lambda}^{\Lambda} \int_{-\Lambda}^{\Lambda} \frac{dk_1}{2\pi} \frac{dk_2}{2\pi} J_{ij} \vec{S} \cdot c_{i\alpha k_1}^\dagger \frac{\vec{\sigma}_{\alpha\beta}}{2} c_{j\beta k_2}, \end{aligned} \quad (2)$$

where the dispersion has been linearized ($E = v_F k$), J_{ij} is a Hermitian matrix, \vec{S} denotes the Pauli matrices, and we are assuming an isotropic spin coupling $J_x = J_y = J_z$ for simplicity.

Next, we consider density-density interactions between the electrons of the form

$$H_{\text{int}} = \frac{1}{2} \int \int dx dy \rho(x) U(x-y) \rho(y), \quad (3)$$

where the density operator ρ is given in terms of the second quantized electron field $\Psi_\alpha(x) = \sum_i \int dk/(2\pi) \Psi_{i\alpha k}(x)$ as $\rho = \Psi_\uparrow^\dagger \Psi_\uparrow + \Psi_\downarrow^\dagger \Psi_\downarrow$. Writing the electron field in terms of outgoing and incoming fields as $\Psi_\alpha(x) = \Psi_{O\alpha}(x) + \Psi_{I\alpha}(x)$, the Hamiltonian in (3) takes the form

$$\begin{aligned} H_{\text{int}} &= \int dx \sum_{\alpha,\beta} [g_1 \Psi_{O\alpha}^\dagger \Psi_{I\beta}^\dagger \Psi_{O\beta} \Psi_{I\alpha} + g_2 \Psi_{O\alpha}^\dagger \Psi_{I\beta}^\dagger \Psi_{I\beta} \Psi_{O\alpha} \\ &+ \frac{1}{2} g_4 (\Psi_{O\alpha}^\dagger \Psi_{O\beta}^\dagger \Psi_{O\beta} \Psi_{O\alpha} + \Psi_{I\alpha}^\dagger \Psi_{I\beta}^\dagger \Psi_{I\beta} \Psi_{I\alpha})]. \end{aligned} \quad (4)$$

The parameters g_1 , g_2 and g_4 satisfy some RG equations [11, 13], and are given by

$$\begin{aligned} g_2(L) &= \tilde{U}(0) - \frac{1}{2} \tilde{U}(2k_F) + \frac{1}{2} \frac{\tilde{U}(2k_F)}{1 + \frac{\tilde{U}(2k_F)}{\pi v_F} \ln L}, \\ g_1(L) &= \frac{\tilde{U}(2k_F)}{1 + \frac{\tilde{U}(2k_F)}{\pi v_F} \ln L}, \quad \text{and} \quad g_4(L) = \tilde{U}(0), \end{aligned} \quad (5)$$

where L denotes the length scale, and \tilde{U} is the Fourier transform of U . (We have ignored umklapp scattering).

The junction S -matrix satisfies an RG equation which was derived in Refs. [11,12] in the absence of Kondo couplings. We find that the Kondo couplings J_{ij} do not affect the RG flows of the S -matrix up to second order in the J_{ij} . Since we are mainly interested in the flows of the J_{ij} , we will assume for simplicity that we are at a FP of the RG equations for S_{ij} . We will study what happens near two particular FPs of S_{ij} .

We use the technique of ‘poor man’s RG’ [14,15] to derive the RG equations for the Kondo couplings J_{ij} . (The details will be presented elsewhere). To second order in the couplings J_{ij} and the parameters g_a (which are given in Eq. (5)), we find that

$$\begin{aligned} \frac{dJ_{ij}}{d\ln L} = \frac{1}{2\pi v_F} \sum_k [J_{ik}J_{kj} + \frac{1}{2} g_2 (S_{ij} J_{ik}S_{ik}^* + S_{ji}^*J_{kj}S_{jk}) \\ - \frac{1}{2}(g_2 - 2g_1) (J_{ik}S_{kk}^*S_{kj} + S_{ki}^*S_{kk}J_{kj})], \end{aligned} \quad (6)$$

We now consider two possibilities for the S -matrix. The first case is that of N disconnected wires for which the S -matrix is given by the $N \times N$ identity matrix (up to phases). We consider a highly symmetric form of the Kondo coupling matrix (consistent with the symmetry of the S -matrix) in which all the diagonal entries are J_1 and all the off-diagonal entries are J_2 , with both J_1 and J_2 being real. Eq. (6) then gives

$$\begin{aligned} \frac{dJ_1}{d\ln L} &= \frac{1}{2\pi v_F} [J_1^2 + (N-1)J_2^2 + 2g_1J_1], \\ \frac{dJ_2}{d\ln L} &= \frac{1}{2\pi v_F} [2J_1J_2 + (N-2)J_2^2 - (g_2 - 2g_1)J_2]. \end{aligned} \quad (7)$$

(For $N = 2$ and $g_1 = 0$, Eq. (7) agrees with the results in Ref. [6]). Since $g_1(L = \infty) = 0$, Eq. (7) has a FP at $(J_1, J_2) = (0, 0)$. If $\nu \equiv g_2(L = \infty)/(2\pi v_F) > 0$ (repulsive interactions), a linear stability analysis shows that this FP is stable to small perturbations in J_2 . For small perturbations in J_1 , this FP is marginal; a second order analysis shows that it is stable if $J_1 < 0$ and unstable if $J_1 > 0$, i.e., it is the usual *ferromagnetic* fixed point which is found for Fermi liquid leads. However, the approach to the fixed point is quite different when the leads are TLLs. At large length scales, the FP is approached as $J_1 \sim -1/\ln L$ and $J_2 \sim 1/L^\nu$. From this, we can deduce the behavior at very low temperatures, namely,

$$J_1 \sim -1/(\ln 1/T), \quad \text{and} \quad J_2 \sim T^\nu. \quad (8)$$

This is in contrast to the behavior of J_2 for Fermi liquid leads, i.e., for $g_1 = g_2 = 0$. In that case, Eq. (7) again gives a FP at $(J_1, J_2) = (0, 0)$, but J_2 approaches zero as $1/(\ln 1/T)^2$. Note that J_2 (which measures the asymmetry between the channels) approaches zero faster than J_1 for Fermi liquid leads; but for TLLs, it goes to zero much faster, i.e., as a power of T .

Fig. 1 shows the RG flows for three wires for $\tilde{U}(0) = \tilde{U}(2k_F) = 0.2(2\pi v_F)$. [This gives a value of ν which is comparable to what is found in several experimental systems (see [16] and references therein). In Figs. 1 and 2, the values of J_{ij} are shown in units of $2\pi v_F$.] We see that the RG flows take a large range of initial conditions to the FP at $(0, 0)$. For all other initial conditions, we see that there are two directions along which the Kondo couplings flow to infinity; these are given by $J_2 = J_1$ and $J_2 = -J_1/(N-1)$ (with $N = 3$). This asymptotic behavior can be understood by analyzing Eq. (7) after ignoring the terms of order g_1 and g_2 .

The second case that we study is called the Griffiths S -matrix. Here all the N wires are connected to each other and there is maximal transmission, subject to the constraint that

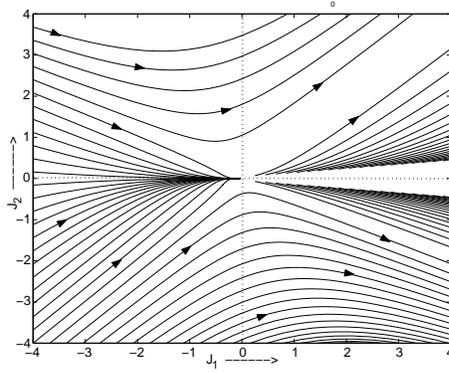


Fig. 1 – RG flows of the Kondo couplings for three disconnected wires, with $\tilde{U}(0) = \tilde{U}(2k_F) = 0.2(2\pi v_F)$.

there is complete symmetry between the wires. The resultant S -matrix has, up to phases, all the diagonal entries equal to $-1 + 2/N$ and all the off-diagonal entries equal to $2/N$. We again consider a highly symmetric form of the Kondo coupling matrix, with real parameters J_1 and J_2 as the diagonal and off-diagonal entries respectively. Eq. (6) then gives

$$\begin{aligned} \frac{dJ_1}{d \ln L} &= \frac{1}{2\pi v_F} [J_1^2 + (N-1)J_2^2 + 2g_1(1 - \frac{2}{N})^2 J_1 - 4g_1(1 - \frac{2}{N})(1 - \frac{1}{N}) J_2], \\ \frac{dJ_2}{d \ln L} &= \frac{1}{2\pi v_F} [2J_1J_2 + (N-2)J_2^2 - \frac{4g_1}{N}(1 - \frac{2}{N}) J_1 + (g_2 - 2g_1(1 - \frac{2}{N})^2) J_2]. \end{aligned} \quad (9)$$

(For $N = 2$ and $g_1 = 0$, Eq. (9) agrees with the results in Ref. [5]). Eq. (9) has a FP at the origin (which is unstable for $g_2(\infty) > 0$), and two strong coupling FPs as before. Fig. 2 shows a picture of the RG flows for three wires. The couplings are again seen to flow to infinity along one of the two directions $J_2 = J_1$ and $J_2 = -J_1/(N-1)$.

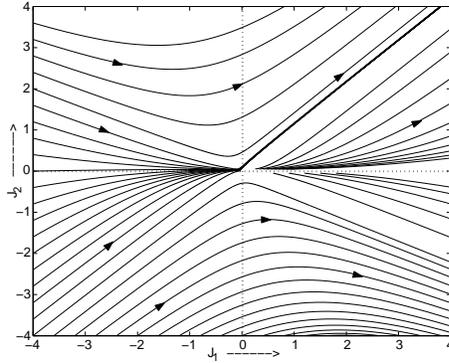


Fig. 2 – RG flows of the Kondo couplings for the Griffiths S -matrix for three wires, with $\tilde{U}(0) = \tilde{U}(2k_F) = 0.2(2\pi v_F)$.

We will now see how the different S -matrices and RG flows discussed above can be interpreted in terms of lattice models as was done for the two-wire case in Ref. [5]. The case of N

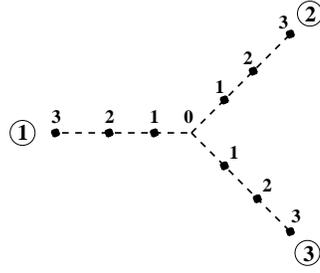


Fig. 3 – A lattice model for the S -matrices discussed in the text.

disconnected wires can be realized by the lattice model shown in Fig. 3. The Hamiltonian is taken to be of the tight-binding form, with a hopping amplitude $-t$ on all the bonds, except on the N bonds connecting to the junction site $n = 0$ where they are taken to be zero. (This is equivalent to removing the junction site from the system). The impurity spin is coupled to the sites $n = 1$ on the different wires by the Hamiltonian

$$H_{\text{spin}} = F_1 \vec{S} \cdot \sum_i \sum_{\alpha, \beta} \Psi_{\alpha}^{\dagger}(i, 1) \frac{\vec{\sigma}_{\alpha\beta}}{2} \Psi_{\beta}(i, 1) + F_2 \vec{S} \cdot \sum_{i \neq j} \sum_{\alpha, \beta} \Psi_{\alpha}^{\dagger}(i, 1) \frac{\vec{\sigma}_{\alpha\beta}}{2} \Psi_{\beta}(j, 1), \quad (10)$$

where $\Psi_{\alpha}(i, 1)$ denotes the second quantized electron field at site 1 on wire i . (Eq. (15) below will provide a justification for this Hamiltonian). In Eq. (10), F_1 and F_2 denote amplitudes for spin-dependent scattering from the impurity within the same wire and between two different wires respectively. Now, we find that the Kondo coupling matrix J_{ij} in Eq. (2) is as follows: all the diagonal entries are given by J_1 and all the off-diagonal entries are given by J_2 , where

$$J_1 = 4F_1 \sin^2 k_F, \quad \text{and} \quad J_2 = 4F_2 \sin^2 k_F \quad (11)$$

for modes with redefined wave numbers lying close to zero. The RG flow of this is given in Eq. (7). In particular, the approach to the FP at $(J_1, J_2) = (0, 0)$ given by Eq. (8) at low temperatures implies that spin-flip scattering within the same wire or between two different wires will have quite different temperature dependences.

The case of the Griffiths S -matrix can also be realized by the lattice shown in Fig. 3 and a tight-binding Hamiltonian. The hopping amplitude is now $-t$ on all bonds, except for the N bonds connecting to the junction site where it is taken to be $t_1 = -t\sqrt{2/N}$. We then find that the S -matrix is of the Griffiths form for all values of the wave number k . The impurity spin is then coupled to the junction site and the $n = 1$ sites by the Hamiltonian

$$H_{\text{spin}} = F_3 \vec{S} \cdot \sum_{\alpha, \beta} \Psi_{\alpha}^{\dagger}(0) \frac{\vec{\sigma}_{\alpha\beta}}{2} \Psi_{\beta}(0) + F_4 \vec{S} \cdot \sum_i \sum_{\alpha, \beta} \Psi_{\alpha}^{\dagger}(i, 1) \frac{\vec{\sigma}_{\alpha\beta}}{2} \Psi_{\beta}(i, 1), \quad (12)$$

where $\Psi_{\alpha}(0)$ denotes the electron field at the junction site with spin α . Then the Kondo coupling matrix J_{ij} in Eq. (2) has all the diagonal entries equal to J_1 and all the off-diagonal entries equal to J_2 , where

$$J_1 = \frac{4F_3}{N^2} + 2F_4 \left[1 - \left(1 - \frac{2}{N}\right) \cos 2k_F \right], \quad \text{and} \quad J_2 = \frac{4F_3}{N^2} + \frac{4F_4}{N} \cos 2k_F \quad (13)$$

for modes with wave numbers lying close to zero. Eq. (9) gives the RG flows of these parameters. Eq. (13) implies

$$J_1 - J_2 = 2F_4 (1 - \cos 2k_F), \quad \text{and} \quad J_1 + (N-1)J_2 = \frac{4F_3}{N} + 2F_4 (1 + \cos 2k_F). \quad (14)$$

Since $0 < k_F < \pi$, $1 \pm \cos 2k_F$ lie between 0 and 2. In the first quadrant of Fig. 2, we see that $J_1 + (N-1)J_2$ goes to ∞ much faster than $|J_1 - J_2|$; Eq. (14) then implies that F_3 goes to ∞ and $|F_4| \ll F_3$. In the fourth quadrant of Fig. 2, $J_1 - J_2$ goes to ∞ much faster than $J_1 + (N-1)J_2$; hence F_4 goes to ∞ and F_3 goes to $-\infty$. These flows to strong coupling have the following physical interpretations. In the first case, F_3 flows to ∞ which means that the impurity spin is strongly and antiferromagnetically coupled to an electron spin at the junction site $n = 0$; hence those two spins will combine to form an effective spin of $\mathcal{S} - 1/2$. In the second case, the impurity spin is coupled strongly and ferromagnetically to an electron spin at the site $n = 0$, and strongly and antiferromagnetically to electron spins at the sites $n = 1$ on each of the N wires to form an effective spin of $\mathcal{S} + 1/2 - N/2$.

We considered above two kinds of S -matrices and found that the Kondo couplings flow to infinity for many initial conditions. We will now show through an example that the vicinity of a strong coupling FP can be studied through an expansion in the inverse of the Kondo coupling [15]. Following the discussion after Eq. (14), let us assume that the RG flows for the case of the Griffiths S -matrix have taken us to a strong coupling FP along the line $J_2 = J_1$; thus the impurity spin is coupled to the electron spin at $n = 0$ with a large and positive (antiferromagnetic) value F_3 , while its couplings to the sites $n = 1$ have the value $F_4 = 0$. (The arguments given below do not change significantly if $F_4 \neq 0$, provided that $|F_4| \ll F_3$). From the first term in Eq. (12), we see that the impurity spin couples to an electron at $n = 0$ to form an effective spin of $\mathcal{S} - 1/2$; the energy of this spin state is $-F_3(\mathcal{S} + 1)/2$. This lies far below the high energy states in which an electron at site $n = 0$ forms a total spin of $\mathcal{S} + 1/2$ with the impurity spin (these states have energy $F_3\mathcal{S}/2$), or the states in which the site $n = 0$ is empty or doubly occupied (these states have zero energy).

We now perturb in $1/F_3$. The unperturbed Hamiltonian corresponds to N disconnected wires along with the impurity spin coupled to the junction site $n = 0$. The perturbation H_{pert} consists of the hopping amplitude t_1 on the N bonds connecting the sites $n = 1$ to the junction site. Using this perturbation, we can find an effective Hamiltonian [15]. If $\mathcal{S} > 1/2$, we find that the effective Hamiltonian has no terms of order t_1 , and is given by

$$H_{\text{eff}} = F_{1,\text{eff}} \vec{\mathcal{S}}_{\text{eff}} \cdot \sum_i \sum_{\alpha,\beta} \Psi_{\alpha}^{\dagger}(i,1) \frac{\vec{\sigma}_{\alpha\beta}}{2} \Psi_{\beta}(i,1) + F_{2,\text{eff}} \vec{\mathcal{S}}_{\text{eff}} \cdot \sum_{i \neq j} \sum_{\alpha,\beta} \Psi_{\alpha}^{\dagger}(i,1) \frac{\vec{\sigma}_{\alpha\beta}}{2} \Psi_{\beta}(j,1)$$

with $F_{1,\text{eff}} = F_{2,\text{eff}} = -\frac{8t_1^2}{F_3(\mathcal{S}+1)(2\mathcal{S}+1)}$, (15)

and $\vec{\mathcal{S}}_{\text{eff}}$ denotes an object with spin $\mathcal{S} - 1/2$. We thus find a weak interaction between $\vec{\mathcal{S}}_{\text{eff}}$ and all the sites labeled as $n = 1$ in Fig. 3. [If the impurity has $\mathcal{S} = 1/2$, the electron at $n = 0$ forms a singlet with the impurity.] For $\mathcal{S} > 1/2$, we see that Eq. (15) has the same form as in Eqs. (10-11), where the effective couplings $J_{1,\text{eff}} = 4F_{1,\text{eff}} \sin^2 k_F$ and $J_{2,\text{eff}} = 4F_{2,\text{eff}} \sin^2 k_F$ are equal, negative and small. With these initial conditions, we see from Eq. (7) and Fig. 1 that the Kondo couplings flow to the FP at $(J_{1,\text{eff}}, J_{2,\text{eff}}) = (0, 0)$.

We thus obtain a picture of the RG flows at both short and large length scales. We start with the Griffiths S -matrix with certain values of the Kondo couplings, and finally end at the multi-channel FP of the disconnected S -matrix. But in this letter, we have restricted

ourselves to weak inter-electron interactions, since we use perturbative methods to analyze the effects of g_i . Hence, the Luttinger parameter K is restricted to be less than but close to unity. An interesting question to address is whether this analysis is true for strong inter-electron interactions when K is much less than unity. In the two-wire case, it was shown that at strong interactions, it is the two-channel *antiferromagnetic* Kondo fixed point which is stabilized for $K < 1/2$ [6]. An equivalent analysis for N wires is required [17].

To summarize, we have studied the Kondo effect at a junction of N quantum wires and find an interesting interplay of the Kondo logarithms and the TLL power laws. We find that the scaling of the Kondo couplings depends on the S -matrix at the junction. For the case of disconnected wires and repulsive interactions, there is a range of Kondo couplings which flow towards a multi-channel FP at $(J_1, J_2) = (0, 0)$. At low temperatures, we find spin-flip scattering processes with temperature dependences which are dictated by both the Kondo effect and the inter-electron interactions. It may be possible to observe such scatterings by placing a quantum dot with a spin at a junction of several wires with interacting electrons. At the fully connected or Griffiths S -matrix, we find that the Kondo couplings flow to a strong coupling FP, where their fate is decided by a $1/J$ analysis. There is a range of initial conditions which again lead to the FP at $(J_{1,\text{eff}}, J_{2,\text{eff}}) = (0, 0)$.

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