

# AC-conductance of a quantum wire with electron-electron interaction

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The complex ac-response of a quasi-one dimensional electron system in the one-band approximation with an interaction potential of finite range is investigated. It is shown that linear response is exact for this model. The influence of the screening of the electric field is discussed. The complex absorptive conductance is analyzed in terms of resistive, capacitive and inductive behaviors.

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## I. INTRODUCTION

Experimental and theoretical investigations of the ac-transport in nano-structures are of profound scientific interest since they provide insight into the behavior of (open) quantum systems in non-equilibrium that are externally controllable within wide ranges of parameters. In addition, possible applications of nano-structures in future electronic devices, which will have to operate at very high frequencies, require detailed knowledge of their frequency and time dependent transport behavior.

Electron transport in nano-structures is very strongly influenced by charging effects. Most striking is the Coulomb-blockade<sup>1</sup> of the dc-current through tiny tunnel junctions when the bias voltage and the temperature are smaller than the “charging energy”,  $E_C = e^2/2C$  ( $e$  elementary charge,  $C$  “capacitance of the tunnel junction”). The use of a capacitance has been justified by observing that its values determined from  $E_C$  are consistent with those obtained from the geometry of the junction<sup>2,3</sup>;  $C$  was found to be of the order of  $10^{-15}$ F and much smaller for metallic junctions<sup>4</sup>. Interactions also dominate transport through islands between two tunnel contacts in series in a semiconductor quantum wire. The *linear* conductance shows pronounced peaks<sup>5</sup> if the external chemical potential coincides with the difference between the ground state energies of  $N + 1$  and  $N$  electrons in the island. In the charging model, these energies are again given in terms of a “capacitance  $C$ ”,  $E(N) = N^2 e^2 / 2C$ ,  $C \approx 10^{-15}$ F. One can ask, how small a capacitance can be without being influenced by quantum effects. The limitations of the charging model become obvious in the non-linear transport properties: fine-structure in the current-voltage characteristic is related to the quantum properties of the interacting electrons<sup>6,7,8,9</sup>.

In recent years, frequency dependent electrical response of systems with reduced dimensionality became the subject of activities. These techniques are of particular interest since no current and voltage probes have to be attached to the sample. The current response to microwave and far-infrared radiation on the transport through semiconductor microstructures has been

studied<sup>10,11,12,13</sup>. It has been found that the absorption of microwaves leads to a characteristic broadening of the conductance peaks of semiconductor quantum dots in the Coulomb-blockade region<sup>14,15</sup>. Infrared absorption<sup>16,17,18</sup> of quantum dots and wires provided mainly information on the parts of the excitation spectrum that are only weakly influenced by the interaction due to Kohn’s theorem. However, Raman scattering from quantum wires and dots showed signatures of the dispersion of the collective excitations<sup>19,20,21</sup>. The absorption of microwaves in an ensemble of metallic grains has been investigated experimentally and theoretically in many papers<sup>22,23</sup>.

Absorption and scattering of electromagnetic radiation are only one possibility to measure ac-transport properties without applying voltage and current probes that may disturb the system’s properties severely. More recently, other highly sophisticated, non-invasive techniques for determining ac-conductances have been pioneered<sup>24</sup>. Coupling a system of about  $10^5$  mesoscopic rings to a highly sensitive superconducting micro-resonator the perturbation of the resonance frequencies and quality factors has been used to determine the real and imaginary parts of an ac-conductance. Here, the fundamental question arises, what the differences between the “conductances” as determined by different methods are<sup>25</sup>.

Theoretical approaches have been developed, ranging from semi-classical rate equation approximations to fully quantum mechanical attempts. The linear theory of ac-quantum transport has been restricted to non-interacting systems<sup>26,27,28,29,30</sup>. How to define quantum capacitances<sup>28</sup> and inductances<sup>27,29</sup> was addressed. The ac-transport through mesoscopic structures in the presence of Coulomb interaction was considered by using a self-consistent mean field method<sup>31,32</sup>. This approach strongly relies on the presence of “reservoirs”, “contacts” and “electro-chemical potentials” which are not necessary ingredients of high frequency experiments such as the absorption of microwaves.

Photo-induced transport through a tunnel barrier<sup>33</sup> and tunneling through semiconductor double-barrier

structures<sup>34</sup> has been considered. Charging effects in small semiconductor quantum dots in the presence of time-varying fields were treated<sup>30,35</sup>. The influence of high frequency electric fields on the linear and non-linear transport through a quantum dot with infinitely strong Coulomb repulsion<sup>36</sup> was studied. Photon-assisted tunneling through a double quantum dot has been investigated by using the Keldish technique<sup>37</sup>. The photo-induced transport through a single tunnel barrier in a 1D interacting electron system has been investigated<sup>38</sup>. In most of the latter works, quantum effects of the interaction have been treated only approximatively.

In view of the importance of the interaction for the ac-properties of nano-structures, Luttinger systems are of great interest. Here, the interaction can be taken into account exactly. The conductance of a tunnel barrier in a Luttinger liquid with zero-range interaction has been shown<sup>39,40</sup> to scale with the frequency as  $\omega^{2/g-2}$  ( $g$  interaction parameter, see below). At  $\omega = 0$  repulsive electron-electron interaction suppresses tunneling completely, even for a vanishingly small potential barrier. However, it has been also shown that for  $\omega \rightarrow 0$  there is a *displacement contribution* to the current which can dominate the transport for very strong repulsive interaction and very high barrier<sup>41</sup>.

The driving voltage has been *assumed ad hoc* in most of these works. Since the driving electric field is determined by the interaction between the electrons<sup>42,43</sup>, the current can be expected to depend on how charges induced by an external electric field are rearranged by the interaction. Even in the limit of dc-transport through a Luttinger system this has been argued to be so<sup>44,45</sup>. The dependence of the ac-properties of a tunnel barrier in Luttinger system on the shape of the electric driving field has been investigated<sup>42</sup>. It has been found that the current depends only on the integral over the driving electric field – the external voltage – only in the dc-limit, even in the non-linear regime. One of the side results was to confirm that linear response is exact for the ideal Luttinger system<sup>42,46,47</sup>. The ac-properties of the Luttinger liquid with spatially varying interaction strength were also studied<sup>46,47,48</sup>.

In this paper, we concentrate on the ac-transport properties of electrons described by the Luttinger model with an interaction of finite range. The model is exactly solvable. Since its current response can be determined without approximations, answers to fundamental questions can be found, such as (i) to identify the specific signatures of the electron-electron interaction in ac-transport, (ii) to understand the influence of the properties of the driving electric field, how to define conductances, and (iv) to understand ac-transport in terms of resistive, capacitive and inductive behaviors. The latter yield quantum analogs of impedances that are common in classical electrodynamics. Such parameters are also often used for describing the transport in nanostructures. Therefore, a quantum approach towards their definition is highly desirable. However, in the quantum regime, they depend

not only on the interaction, but also on the frequency and the shape of the applied electric field. Together with their microscopic definitions, it is thus important to determine the range of parameters for their validity.

The interaction potential is obtained by using a 3D screened Coulomb potential (screening length  $\alpha^{-1}$ ) and projecting to a quasi-1D quantum wire of a finite width  $d$ . When  $\alpha^{-1} \ll d$  we recover the Luttinger liquid with zero-range interaction. On the other hand, when  $\alpha^{-1} \gg d$  we obtain the 1D-analog of the Coulomb interaction. The excitation spectrum of this model shows an inflection point at a frequency  $\omega_p$  that increases monotonically with the interaction strength, and with the inverse of a characteristic length associated with the interaction<sup>49</sup>.

By using linear response theory we obtain the complex, frequency-dependent non-local conductivity given by the current-current correlation function. It contains the dispersion relation of the elementary excitations, and turns out to be independent of the temperature within the model. There are no non-linear contributions to the current in this model.

The conductivity describes the current response of the system with respect to an electric field  $E(x', \omega)$ . In an experiment, either the current as a function of an *external voltage* (in the dc-limit), or the absorption of electromagnetic energy at frequency  $\omega$  from an external field is determined. In both cases, not too much is known about the internal field. Therefore, it is reasonable to search for quantities that do not depend on the spatial form of the field. One possibility is<sup>50</sup> to define the conductance  $\Gamma_1$  by using the absorbed power. One finds that, in the dc-limit, the result is indeed independent of the shape of the field. However, in ac-transport, the shape of the square of the Fourier transform of the electric field appears as a multiplicative factor. The remaining factor is the density of collective excitations of the interacting electrons. It has a resonance at  $\omega_p$ . For non-interacting electrons  $dq/d\omega = v_F^{-1}$  and  $\Gamma_1(\omega)$  reveals merely the structure of the electric field.

That the ac-conductance depends on the shape of the driving electric field leads to the question what the nature of this field is in an interacting system. We show that for the conductivity of an ideal Luttinger liquid, it is the *external electric* field which has to be used since linear response is exact. Furthermore, we will demonstrate that this is also true for the absorptive conductance.

Having determined the absorptive part of the conductance,  $\Gamma_1(\omega)$ , the reactive part  $\Gamma_2(\omega)$  may be obtained by Kramers-Kronig transformation. The complex conductance  $\Gamma(\omega) = \Gamma_1(\omega) + i\Gamma_2(\omega)$  relates the average current with the voltage. The current as a function of time consists of two parts. One,  $\propto \Gamma_1$ , is in phase with the driving field. The second,  $\propto \Gamma_2$ , is phase shifted by  $\pm\pi/2$ .

When  $\omega$  is small, we can expand

$$\Gamma_1(\omega) = g \frac{e^2}{h} + \gamma_1 \omega^2 + O(\omega^4). \quad (1)$$

The first term corresponds to the quantized contact

conductance<sup>51</sup>  $R_K^{-1} = e^2/h$ . It is here renormalized by the interaction parameter<sup>39</sup>  $g$ . The term  $\propto \omega^2$  indicates whether the current is capacitive ( $\gamma_1 > 0$ ) or inductive ( $\gamma_1 < 0$ ). For  $\Gamma_2$  we find

$$\Gamma_2(\omega) = \omega\gamma_2 + O(\omega^3). \quad (2)$$

This quantity also indicates if the system behaves capacitively and inductively,  $\gamma_2 < 0$  and  $\gamma_2 > 0$ , respectively. However, in the latter case,  $\gamma_1$  can still be positive indicating capacitive behavior of the real part of the conductance.

For frequencies close to the resonance, the Kramers-Kronig transformation gives

$$\Gamma_2(\omega) \approx \gamma_m(\omega - \omega_m^*), \quad (3)$$

with  $\gamma_m > 0$  when  $\gamma_2 < 0$  and  $\omega_m^* \approx \omega_m$  (position of maximum of  $\Gamma_1(\omega)$ ). The quantity  $\gamma_m$  indicates capacitive and/or inductive behavior close to the resonance frequency ( $\approx \omega_p$ ).

The height or, equivalently, the width of the resonance in  $\Gamma_1$  defines also a resistance,  $R$ . In contrast to the contact resistance, it is truly “dissipative” and related to the pair excitations of the Luttinger liquid. It is also contained in  $\gamma_1$ , though its numerical value for small  $\omega$  is different from the one near the resonance. Generally, we find that it is only possible to define resistances, capacitances and inductances in certain limited parameter regions<sup>49</sup>.

In order to observe capacitive behavior, the interaction between the electrons should be *sufficiently long range*. This is consistent with the results of a different approach in which Coulomb blockade behavior at a tunnel barrier between two Luttinger liquids has been discussed<sup>52</sup>. Also there, non-zero range of the interaction is necessary for capacitive behavior.

In the next section, we describe briefly the model and the dispersion relations for various interaction potentials. We calculate the ac-conductance and study external versus internal driving fields in the section 3. Section 4 contains the identification of quantum impedances. Section 5 contains the discussion of the results.

## II. LUTTINGER LIQUID WITH LONG-RANGE INTERACTION

### A. Outline of the model

The Luttinger liquid is a model for the low-energy excitations of a 1D interacting electron gas<sup>53,54,55,56</sup>. Its excitation spectrum can be calculated analytically. Also many of the thermodynamical and the transport properties, as the linear conductivity, can be determined, even in the presence of perturbing potentials. The main assumption is the linearization of the free-electron dispersion relation near the Fermi level. The starting point is

the Hamiltonian for interacting Fermions with, say, periodic boundary conditions<sup>57</sup>

$$H = \hbar v_F \sum_{k,s=\pm} (sk - k_F)(c_{ks}^\dagger c_{ks} - \langle c_{ks}^\dagger c_{ks} \rangle_0) + \sum_{k_1,s_1 \dots k_4,s_4} V_{k_1,s_1 \dots k_4,s_4} c_{k_1 s_1}^\dagger c_{k_2 s_2}^\dagger c_{k_3 s_3} c_{k_4 s_4}. \quad (4)$$

Here,  $c_{ks}^\dagger$ ,  $c_{ks}$  are the creation and annihilation operators for Fermions in the states  $|ks\rangle$  of wave number  $k = 2\pi n/L$  ( $n = 0, \pm 1, \pm 2, \dots$ ) in the branches  $s = \pm$ ,  $k_F$  the Fermi wave number,  $V$  the Fourier-transform of the electron-electron interaction, and  $\langle \dots \rangle_0$  denotes an average in the ground state.

Formally, the Fermion Hamiltonian can be transformed into a Bosonized form. For spinless particles with an interaction that depends only on the distance between the particles,  $V(|x - x'|)$ , and taking into account only forward scattering, one obtains a bilinear form in the Boson operators which can be diagonalized by a Bogolubov transformation<sup>56</sup>. The result is

$$H = \sum_q \hbar\omega(q) b_q^\dagger b_q. \quad (5)$$

The spectrum of the pair excitations corresponding to the Bosonic creation and annihilation operators  $b_q^\dagger$ ,  $b_q$  is given by the Fourier transform of the interaction potential<sup>58</sup>  $V(q)$ ,

$$\omega(q) = v_F |q| \sqrt{1 + \frac{V(q)}{\hbar\pi v_F}}. \quad (6)$$

The particle excitations which change the total electron number are omitted here. The number of particles is assumed to be constant,  $N_0 = k_F L/\pi$ . The dispersion relation interpolates between the limit of zero-range interaction ( $q \rightarrow 0$ ) where  $\omega(k) = v |q|$  with the “charge sound velocity”  $v \equiv v_F/g$ , and the limit of non-interacting particles ( $q \rightarrow \infty$ ),  $\omega(q) = v_F |q|$ . The strength of the interaction is defined as

$$\frac{1}{g^2} \equiv 1 + \frac{V(q=0)}{\hbar\pi v_F}. \quad (7)$$

Non-interacting Fermions correspond to  $g = 1$ , repulsive interaction to  $g < 1$ .

The particle density  $\rho(x)$  can be written in terms of the phase variable of the Luttinger model

$$\vartheta(x) = i \sum_{q \neq 0} \text{sgn}(q) \sqrt{\frac{1}{2L|q|}} e^{\varphi(q) - iqx} (b_q^\dagger + b_{-q}) \quad (8)$$

where

$$e^{2\varphi(q)} = \frac{v_F |q|}{\omega(q)}. \quad (9)$$

With the mean particle density  $\rho_0 = N_0/L$  we write

$$\rho(x) \equiv \rho_0 + \frac{1}{\sqrt{\pi}} \partial_x \vartheta(x). \quad (10)$$

For later use in the linear response theory, we need the coupling to the driving voltage  $U(x, t)$  and the current density<sup>42</sup>. The former is given by

$$H_U = e \int_{-\infty}^{\infty} dx \rho(x) U(x, t). \quad (11)$$

The electric field is  $E(x, t) = -\partial_x U(x, t)$ . The external voltage is assumed to be given by  $\int_{-\infty}^{\infty} dx E(x, t) \equiv U(t)$ . The current operator is defined by using the 1D continuity equation for the Heisenberg representation of the operators,

$$J(x, t) \equiv -\frac{e}{\sqrt{\pi}} \dot{\vartheta}(x, t). \quad (12)$$

## B. The interaction potential

In order to obtain the dispersion relation explicitly, we need a specific model for the interaction. Since we want eventually to draw some conclusions on quantum wires we start from a 3D screened Coulomb interaction

$$V(r) = V_0 \frac{e^{-\alpha r}}{r}, \quad (13)$$

with  $V_0 = e^2/4\pi\epsilon\epsilon_0$  and project on the quasi-1D states of the quantum wire.

For the latter, we assume a parabolic confining potential in the  $y$ - and  $z$ -directions independent of  $x$ . The corresponding states are ( $\zeta = \sqrt{y^2 + z^2}$ ),

$$\psi_k(x, \zeta) = \frac{e^{ikx}}{\sqrt{L}} \varphi(\zeta). \quad (14)$$

In the following, we assume for the confining wave function

$$\varphi(\zeta) = \sqrt{\frac{2}{\pi d^2}} e^{-\frac{\zeta^2}{d^2}}, \quad (15)$$

where  $d$  represents the ‘‘diameter’’ of the wire.

We obtain the effective interaction potential for the motion in the  $x$ -direction from the matrix elements of (13) in the state (15) by performing the integrations with respect to  $y$  and  $z$ ,

$$V(x) = -\frac{2V_0}{\alpha d^2} \int_0^{\infty} d\zeta e^{-\frac{\zeta^2}{d^2}} \frac{d}{d\zeta} \left[ e^{-\sqrt{\alpha^2 x^2 + \zeta^2}} \right]. \quad (16)$$

Its Fourier transform is

$$V(q) = V_0 e^{\frac{d^2}{4}(q^2 + \alpha^2)} E_1 \left( \frac{d^2}{4} (q^2 + \alpha^2) \right). \quad (17)$$

The function  $E_1$  is the exponential integral<sup>59</sup>.

Two limiting cases are of particular interest. When  $z \gg 1$ ,  $E_1(z) \approx \exp(-z)/z$ . Thus, for  $\alpha d \gg 1$ ,

$$V(q) = \frac{4V_0}{d^2} \frac{1}{q^2 + \alpha^2}. \quad (18)$$

This is the Fourier transform of

$$V(x) = \frac{1}{2} V_L \alpha e^{-\alpha|x|}. \quad (19)$$

For  $\alpha \rightarrow \infty$  but with  $V_L \equiv 4V_0/\alpha^2 d^2 = \text{const}$ , this is  $V_L \delta(x)$ , the zero-range interaction with the strength  $V_L$  of the conventional Luttinger liquid.

When  $\alpha d \ll 1$  we still get the above result (18) as long as  $qd \gg 1$  which implies that in  $x$ -space  $V(x \rightarrow 0)$  behaves as (19) but with  $V(x=0) = \sqrt{\pi} V_0/d$ . For  $z \rightarrow 0$ ,  $E_1(z) \approx -\ln z$  so that for  $qd \ll 1$ ,

$$V(q) \approx -V_0 \ln((\alpha^2 + q^2) d^2). \quad (20)$$

This is the same behavior as obtained by starting from the 1D-equivalent of the Coulomb interaction<sup>49,58,60</sup> implying the interaction falls off as  $x^{-1}$  in space.

In many of the quantum wire experiments metallic gates are present in some distance, say  $D$ , from the wire (diameter  $d \ll D$ ). In order to discuss the changes in the interaction induced by the presence of the gates we can consider an infinite metal plate parallel to the wire. This changes the interaction potential according to

$$V_D(r) = V(r) - V(|\vec{r} + \vec{D}|), \quad (21)$$

due to the presence of the mirror charge. It is clear that this influences the results only when  $\alpha^{-1} \geq D$ . Assume then  $\alpha = 0$ . The cutoff of the Coulomb tail of the potential is in this limit given by  $D$  instead of  $\alpha^{-1}$ . The results to be discussed below for  $\alpha d \ll 1$  apply also for this limit with  $\alpha$  replaced by  $D^{-1}$ .

## C. Results for the dispersion law

Results for the dispersion are shown in Fig. 1

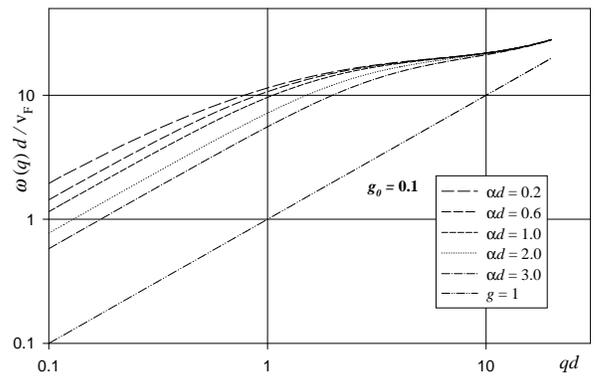


FIG. 1. Double logarithmic plot of the dispersion relation  $\omega(q)$  of the Luttinger model with  $g_0 = 0.1$  and different ranges  $\alpha d$  ( $\alpha$  inverse potential range in position space,  $d$  diameter of quantum wire).

for interaction parameter  $g_0 = 0.1$  ( $V_0 \equiv \hbar\pi v_F(g_0^{-2} - 1)$ ) and various  $\alpha d$ .

There is a crossover between the interacting and the non-interacting limits for  $q \rightarrow 0$  and  $q \rightarrow \infty$ , respectively, at the intermediate wave number  $q_p$  corresponding to the characteristic frequency  $\omega_p$ . It is related to the finite range of the interaction in the wave-number space. For zero-range interaction the dispersion becomes linear, and  $\omega(q) = v_F q/g$ .

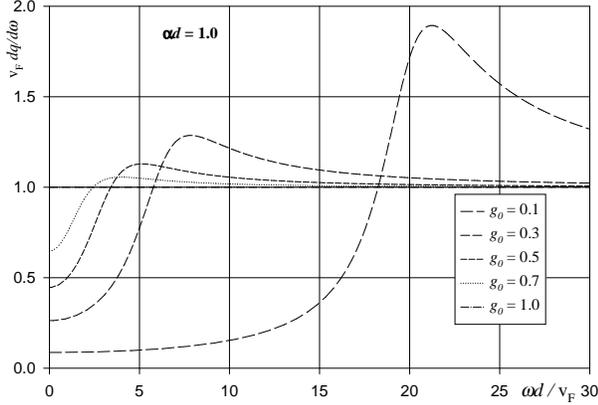


FIG. 2. The density of pair excitations,  $dq/d\omega$  for  $\alpha d = 1$ , different  $g_0$ .

Fig. 2 shows the excitation density  $dq/d\omega$  for various  $g_0$  and  $\alpha d = 1$ .

The frequency  $\omega_p$  and the corresponding wave number  $q_p$  as a function of  $(g_0^{-2} - 1)^{-1}$  are shown in Fig. 3 for various  $\alpha d$ .

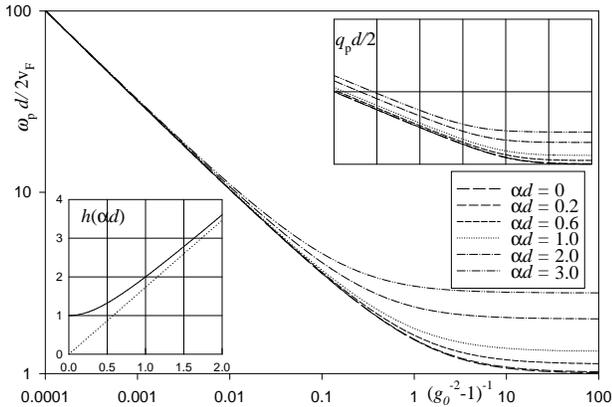


FIG. 3. The resonance frequency  $\omega_p$  and the corresponding wave number,  $q_p$  (insert top right) as a function of the strength of the interaction,  $(g_0^{-2} - 1)^{-1}$  of the Luttinger liquid with interaction of finite range as indicated. Insert bottom left: scaling function  $h(\alpha d)$ .

For a broad range of  $g_0$  the frequency  $\omega_p$  and  $q_p$  decay as  $g_0^{-1}$  and  $g_0^{-1/2}$ , respectively. The data for  $\omega_p$  obey the scaling law ( $\beta_0 \equiv g_0^{-2} - 1$ )

$$\omega_p(\beta_0; \alpha d) = \omega_p(\beta_0 h^{-2}(\alpha d); 0) h(\alpha d). \quad (22)$$

The scaling function  $h(\alpha d)$  is shown in the bottom left insert of Fig. 3 and it is proportional to the limit of  $\omega_p$  for infinitesimally small interaction.

### III. LINEAR RESPONSE

In this section we outline the calculation of the conductance with linear response theory.

#### A. The conductivity

Using the above current, (12), one obtains the complex conductivity,  $\sigma(x - x'; t - t')$  which relates the current at a given point  $x$  at time  $t$  with the driving electric field  $E_{\text{ext}}(x', t')$ ,

$$J(x, t) = \int_{-\infty}^{\infty} dx' \int_{-\infty}^t dt' \sigma(x - x'; t - t') E_{\text{ext}}(x', t'). \quad (23)$$

While the non-locality of the conductivity is unimportant in the dc-limit, it is crucial for time-dependent transport. By assuming that the electric field is concentrated only near a given point, say  $x' = x_0$ , we get

$$J(x, t) = \int_{-\infty}^t dt' \sigma(x - x_0; t - t') U_{\text{ext}}(t') \quad (24)$$

with the voltage drop  $U_{\text{ext}}(t') = \int dx' E_{\text{ext}}(x', t')$  dropping only near  $x_0$ . We see that  $\sigma(x - x_0, t - t')$  plays then the role of a “conductance” that relates the current at some point  $x$  in the system with a voltage dropping at some other point. By using near-field microscopy, one could possibly perform such a non-local experiment. However, it seems to us that it is very hard to measure the current locally in a quantum wire, especially in the region of high frequency.

By Fourier transformation, (23) is equivalent to

$$J(q, \omega) = \sigma(q, \omega) E_{\text{ext}}(q, \omega). \quad (25)$$

The conductivity kernel can be expressed either by the current-current correlation function or, by using the continuity equation, as

$$\sigma(q, \omega) \equiv \sigma_1(q, \omega) + i\sigma_2(q, \omega) = \frac{-i\omega e^2}{q^2} R(q, \omega), \quad (26)$$

with the charge-charge correlation function

$$R(x, t) = -\frac{i}{\hbar} \Theta(t) \langle [\rho(x, t), \rho(0, 0)] \rangle. \quad (27)$$

Here,  $\langle \dots \rangle \equiv \text{Tr}(\exp(-\beta H) \dots) / \text{Tr} \exp(-\beta H)$  is the usual thermal average (temperature  $\propto \beta^{-1}$ ). By using the expression (10) we find the exact result

$$R(q, \omega) = \frac{v_F}{\hbar\pi} \frac{q^2}{\omega^2(q) - (\omega + i0^+)^2}. \quad (28)$$

### B. Analogy with a Brownian particle

A most remarkable feature of the above result becomes transparent by applying the imaginary-time path integral approach. With this, the time-dependent non-linear response to an electric field of arbitrary spatial shape of a Luttinger system was calculated recently<sup>42</sup>. It was found that the average of the phase field (8) obeys the equation of motion of a Brownian particle with mass  $M \rightarrow 0$

$$M\ddot{\vartheta}(x, t) + \int_{-\infty}^t dt' \gamma(t-t') \dot{\vartheta}(x, t') = \mathcal{L}(x, t) \quad (29)$$

subject to an effective external force with the Fourier transform

$$\mathcal{L}(x, \omega) = -\frac{e}{\sqrt{\pi}} \frac{1}{\sigma(x=0, \omega)} \int_{-\infty}^{\infty} dx' E(x', \omega) \sigma(x-x', \omega), \quad (30)$$

and a damping term  $\gamma(t)$  which is given by the non local conductivity. The Fourier transform of  $\gamma(t)$  is

$$\gamma(\omega) = \frac{e^2}{\pi} \frac{1}{\sigma(x=0, \omega)}. \quad (31)$$

From the solution of the equation of motion the current is found,

$$J(x, t) = \int_{-\infty}^{\infty} dx' \int_{-\infty}^t dt' \sigma(x-x', t-t') E(x', t'). \quad (32)$$

Linear response is exact for the Luttinger liquid.

### C. The driving electric field

In this section, we investigate the influence of screening on the response to an external electric field. In particular, we discuss the dc-limit and show that the two limits  $\omega \rightarrow 0$  and  $q \rightarrow 0$  cannot be interchanged. It will turn out that for the Luttinger model, where linear response is exact,

one can use the external field for the calculation of the current. The results will be used to derive absorptive and reactive conductances.

The dielectric response function, which describes the dynamical screening of a charge, is defined as

$$\varepsilon(q, \omega) = \frac{U_{\text{ext}}(q, \omega)}{U_{\text{tot}}(q, \omega)}, \quad (33)$$

where

$$U_{\text{tot}}(q, \omega) = U_{\text{ext}}(q, \omega) + U_{\text{sc}}(q, \omega) \quad (34)$$

is the total potential and  $U_{\text{ext}}$ ,  $U_{\text{sc}}$  are the external and screening potential, respectively. Within the linear screening model, the dielectric response function can be written in terms of the charge-charge correlation function as

$$\frac{1}{\varepsilon(q, \omega)} = 1 - V(q)R(q, \omega). \quad (35)$$

By using the result (28) for  $R(q, \omega)$  we find the explicit expression

$$\varepsilon(q, \omega) = \frac{\omega^2(q) - \omega^2}{\omega_0^2(q) - \omega^2}, \quad (36)$$

with the dispersion of the non-interacting electrons  $\omega_0(q)$ . Equations (33) and (34) imply that

$$\begin{aligned} E_{\text{tot}}(q, \omega) &= E_{\text{ext}}(q, \omega) [1 - V(q)R(q, \omega)] \\ &\equiv E_{\text{ext}}(q, \omega) F(q, \omega). \end{aligned} \quad (37)$$

Using the conductivity (26) with (28) and comparing with (36) we see that

$$\sigma(q, \omega) = \frac{\sigma_0(q, \omega)}{\varepsilon(q, \omega)}. \quad (38)$$

Here,  $\sigma_0$  is the conductivity of the non-interacting electrons. This implies that

$$E_{\text{tot}}(q, \omega) \sigma_0(q, \omega) = E_{\text{ext}}(q, \omega) \sigma(q, \omega). \quad (39)$$

If we calculated the conductivity from the response to the total field, the conductivity would turn out to be that of non-interacting electrons.

This result is true as long as one can use linear screening. It implies also that the voltage drop at frequency  $\omega$  is the same for both fields

$$U_{\text{tot}}(\omega) = \int_{-\infty}^{\infty} dx E_{\text{tot}}(x, \omega) = U_{\text{ext}}(\omega) \quad (40)$$

since for any finite non-zero frequency  $\varepsilon(q \rightarrow 0, \omega) = 1$ .

For a monochromatic external field,

$$E_{\text{ext}}(q, t) = E_{\text{ext}}(q) \cos \omega t, \quad (41)$$

one finds the result

$$E_{\text{tot}}(q, t) = E_{\text{ext}}(q) [\mathcal{R}eF(q, \omega) \cos \omega t + \mathcal{I}mF(q, \omega) \sin \omega t] \quad (42)$$

which means that there is a phase shift between the total and the external field.

A final remark concerns the static limit. While we have for any non-zero frequency  $\varepsilon(q \rightarrow 0, \omega) = 1$ , we find for  $\omega = 0$

$$\lim_{q \rightarrow 0} \varepsilon(q, 0) = \lim_{q \rightarrow 0} \frac{\omega^2(q)}{\omega_0^2(q)} = \frac{1}{g}. \quad (43)$$

By inserting the dispersion relation of the Luttinger model into (28) one obtains

$$E_{\text{tot}}(q) = E_{\text{ext}}(q) \frac{1}{1 + V(q)/\hbar v_F \pi}. \quad (44)$$

The limits  $\omega \rightarrow 0$  and  $q \rightarrow 0$  cannot be inter-changed. The latter result has been used recently, in order to explain that in quantum wires the dc-conductance is not renormalized by the interaction<sup>44,45</sup> (see also (44)). We are discussing here frequency dependent properties. Thus, we always obtain for small frequencies a conductance that is renormalized by the interaction since we consider the limit of infinite system length.

#### D. Absorptive and reactive conductances

Since it is very difficult to detect experimentally the non-local conductivity some average has to be performed. One possibility is to use the absorbed power,  $P(t)$ , in order to define the conductance. This appears to be a natural choice if one wants to describe infrared or microwave experiments,

$$\begin{aligned} P(t) &= \int_{-\infty}^{\infty} dx J(x, t) E_{\text{tot}}(x, t) \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} dq J(q, t) E_{\text{tot}}(-q, t). \end{aligned} \quad (45)$$

We define the average

$$\bar{P} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T dt P(t). \quad (46)$$

Using the Laplace transform

$$P(s) = \int_0^{\infty} dt e^{-st} P(t) \quad (47)$$

we obtain  $\bar{P} = \lim_{s \rightarrow 0} sP(s)$ . The *absorptive conductance* can then be defined by

$$\Gamma_1 = \frac{\bar{P}}{U_{\text{ext}}^2} \quad (48)$$

with  $U_{\text{ext}}(t) = \int dx E_{\text{ext}}(x, t)$ . It is independent of the amplitude of the external field but depends on its shape

in space and time. Physically, it is the absorption constant for electromagnetic radiation. Using (42) one can show that the absorbed power for a monochromatic time dependence is the same for both fields due to the time average. We obtain

$$\bar{P}_\omega = \frac{1}{4\pi} \int_{-\infty}^{\infty} dq \mathcal{R}e\sigma(q, \omega) |E_{\text{ext}}(q)|^2. \quad (49)$$

With (28) we find for the real part of the conductivity

$$\mathcal{R}e\sigma(q, \omega) = \frac{v_F e^2}{2\hbar} [\delta(\omega - \omega(q)) + \delta(\omega + \omega(q))]. \quad (50)$$

This gives the expression

$$\Gamma_1(\omega) = v_F \frac{e^2}{h} L(q(\omega)) \frac{dq}{d\omega} \quad (51)$$

with the Fourier transformed auto-correlation function of the external electric field

$$L(q) \equiv \frac{1}{2U_{\text{ext}}^2} \left| \int_{-\infty}^{\infty} dx e^{iqx} E_{\text{ext}}(x) \right|^2. \quad (52)$$

By a Kramers-Kronig transformation, we can define also a reactive conductance

$$\begin{aligned} \Gamma_2(\omega) &= \frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} d\omega' \frac{\Gamma_1(\omega')}{\omega - \omega'} \\ &= \frac{1}{4\pi U_{\text{ext}}^2} \int_{-\infty}^{\infty} dq \mathcal{I}m\sigma(q, \omega) |E_{\text{ext}}(q)|^2, \end{aligned} \quad (53)$$

it contains information about phase shifts.

For a zero-range interaction the conductance becomes

$$\Gamma_1(\omega) = \frac{g e^2}{h} L\left(\frac{g\omega}{v_F}\right), \quad (54)$$

the same as without interaction<sup>26</sup>, except for the renormalization of the prefactor and the Fermi velocity with the interaction strength  $g$  and  $g^{-1}$ , respectively. In the general case of an interaction potential of finite range we get asymptotically ( $\omega \rightarrow \infty$ )

$$\Gamma_1(\omega) \approx \frac{e^2}{h} L\left(\frac{\omega}{v_F}\right). \quad (55)$$

This reflects that for large  $q$  the dispersion is not influenced by an interaction of a finite range.

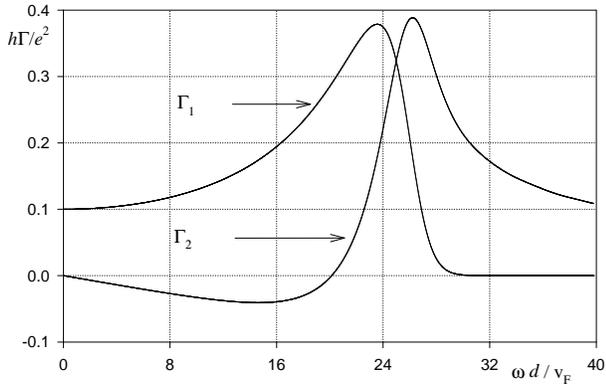


FIG. 4. Real and imaginary part,  $\Gamma_1(\omega)$  and  $\Gamma_2(\omega)$ , of the ac-conductance of a Luttinger wire with finite range interaction;  $\alpha d = 1$ ,  $g = 0.1$ , range of the electric field  $\ell/d = 1/4$ .

A most important feature of the result (51) is the factorization into a part that depends only on the internal properties of the interacting electron system,  $dq/d\omega$ , and a part that contains the information about the shape of the electric field. Only in the limit of vanishing frequency, the shape of the latter is unimportant<sup>26,46</sup>. In general, the ac-response depends on the spatial properties of the electric field<sup>42</sup> which is certainly determined by the interactions.

Most remarkably, the temperature does not enter the result, although  $T \neq 0$  was assumed in the derivation. This is due to the linearization of the spectrum. As long as this assumption is justified, the response of the Luttinger liquid is independent of temperature.

A typical result for  $\Gamma(\omega)$  is shown in Fig. 4. The Fourier transform of the electric field has been assumed to be a Gaussian  $E(x) = E_0 \exp(-2x^2/\ell^2)$ . If the range of the electric field is zero, the zero of  $\Gamma_2(\omega)$ ,  $\omega_m^*$ , and the position of the maximum of  $\Gamma_1(\omega)$ ,  $\omega_m$ , do not agree. However, as soon as  $\ell$  is finite,  $\omega_m \approx \omega_m^*$  for a wide region of parameters.

In the Coulomb case,  $\alpha = 0$ ,  $\Gamma_1 \propto (\ln \omega)^{-1/2}$  for  $\omega \rightarrow 0$ , due to the logarithmic singularity of the dispersion for  $q \rightarrow 0$ .

#### IV. “QUANTUM IMPEDANCES”

In this section, we analyze the results for the complex conductance presented above with respect to “resistive”, “capacitive” and “inductive” behavior. We compare the Luttinger system with an equivalent classical circuit.

##### A. Impedance network

Our system of interacting electrons shows a resonance in the ac-conductance. It can be useful to consider a circuit of capacitances, inductances and resistances, in order

to simulate the frequency behavior. The circuit shown in Fig. 5 contains the minimum set of elements that are necessary for reproducing both the low-frequency behavior and near the resonance.

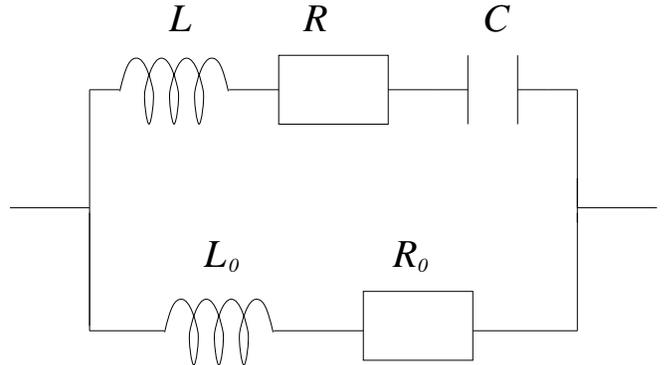


FIG. 5. Classical circuit for simulating the frequency behavior of the complex conductance.

Its complex impedance  $Z(\omega)$  is given by

$$Z^{-1}(\omega) = \frac{i\omega C}{1 + i\omega RC - \omega^2 LC} + \frac{1}{R_0 + i\omega L_0}. \quad (56)$$

The resistance  $R_0$  is fixed to be the resistance at zero frequency,  $h/ge^2$ . The circuit shows a resonance near  $\omega_0 = (LC)^{-1/2}$  with a width depending on  $R$ .

At low frequency, the real and imaginary parts of  $Z^{-1}(-\omega)$ ,  $\Gamma_1(\omega)$  and  $\Gamma_2(\omega)$ , respectively, behave as

$$\Gamma_1(\omega) = R_0^{-1} + \gamma_1 \omega^2, \quad \Gamma_2(\omega) = \gamma_2 \omega \quad (57)$$

with

$$\gamma_1 = RC^2 - \frac{L_0^2}{R_0^3}, \quad \gamma_2 = -C + \frac{L_0}{R_0^2}. \quad (58)$$

The circuit is defined to behave “capacitively” if simultaneously  $\gamma_1 > 0$  and  $\gamma_2 < 0$ . If simultaneously  $\gamma_1 < 0$  and  $\gamma_2 > 0$  the circuit is clearly “inductive”. If  $\gamma_2 = 0$ , i.e. there is no phase shift between current and voltage,  $\gamma_1$  indicates capacitive or inductive behavior depending on whether  $R/R_0$  is larger or smaller than 1, respectively. Note that until  $O(\omega^2)$  the inductance  $L$  does not play any role. Whether the circuit behaves capacitively or inductively near the resonance is therefore to a certain extent independent of its behavior at small frequency.

##### B. Low frequency

We have extracted the parameters  $\gamma_1$  and  $\gamma_2$  (cf. (1) and (2)) which characterize the low-frequency behavior of the conductance from the ac-conductance of the Luttinger liquid.

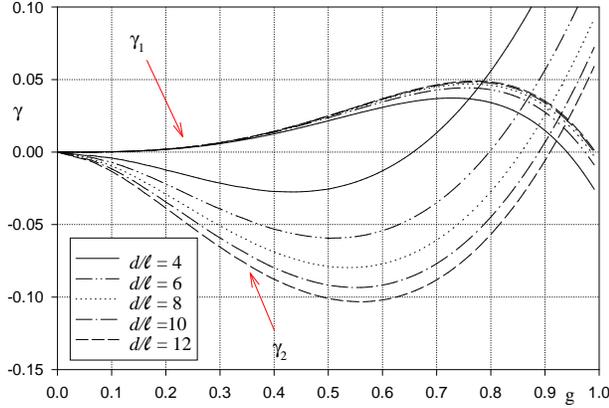


FIG. 6. The parameters  $\gamma_1$  and  $\gamma_2$  (in units of  $e^2 d^2 / h v_F^2$  and  $e^2 d / h v_F$ , respectively) which characterize the low frequency behavior of the ac-conductance of the Luttinger liquid as a function of the interaction parameter  $g$  for different ranges of the electric field,  $\ell$ .

By assuming, as above, the electric field to be the Gaussian distributed, we obtain the explicit results (in units of  $e^2/h$  with  $V(0) = V(q=0)$ )

$$\gamma_1 = \frac{g^3 d^2}{4v_F^2} \left[ \frac{3(1-g^2)}{2} \left( \frac{4V_0}{\alpha^2 d^2 V(0)} - 1 \right) - \frac{\ell^2}{d^2} \right] \quad (59)$$

$$\gamma_2 = -\frac{2v_F}{\pi} \int_0^\infty dq e^{-q^2 \ell^2/4} \left[ \frac{1}{\omega^2(q)} - \frac{1}{\omega_g^2(q)} \right] + \frac{g^2 \ell}{v_F \sqrt{\pi}}$$

where  $\omega_g(q) = v_F |q|/g$ . Figure 6 show  $\gamma_1$  and  $\gamma_2$  as a function of  $g$ . Depending on the range of the field,  $\ell$ , the behavior changes from capacitive (small  $\ell$ ,  $\gamma_1 > 0$ ,  $\gamma_2 < 0$ ) to inductive (large  $\ell$ ,  $\gamma_1 < 0$ ,  $\gamma_2 > 0$ ). As functions of  $g$ ,  $\gamma_1$  and  $\gamma_2$  change also signs. Always, this change of sign happens at a smaller value of  $g$  for  $\gamma_2$ . The “phase separation” lines defined by  $\gamma_1(\ell, g_1) = \gamma_2(\ell, g_2) = 0$  are shown in Fig. 7 for  $\alpha d = 1$ .

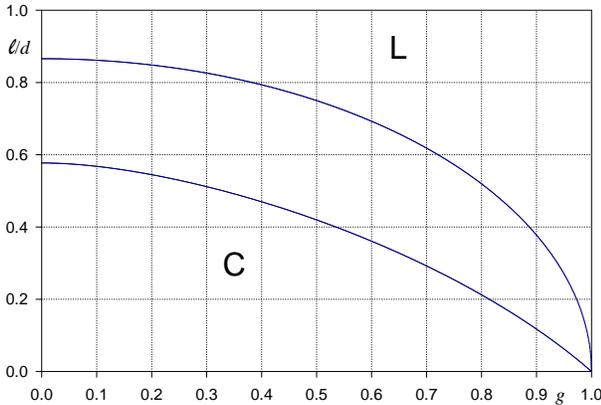


FIG. 7. “Phase trajectories” in the  $(\ell/d)$ - $g$  plane separating capacitive from inductive behaviors of the Luttinger liquid for  $\alpha d = 1$ .

By decreasing  $\alpha d$ , i.e. increasing the range of the interaction, the end points of the two trajectories at  $g = 0$  are shifted to higher values of  $\ell/d$ . The region of capacitive-like behavior increases at the expense of the inductive region for increasing interaction strength.

From the behaviors deep in the capacitive and inductive regions, one can deduct formulas for equivalent inductances and capacitances. However, these are not always unique, although their scaling properties with the parameters of the system are. For instance, in the region denoted by “L” in Fig. 7  $\gamma_2$  is given by the second term in (59) only. This leads, by comparing with (58), to define  $L_0 = h\ell/\sqrt{\pi}e^2v_F$ , independently of the interaction parameter. Qualitatively the same result is obtained when using the expression for  $\gamma_1$ . Apart from a different numerical prefactor, the scaling of the inductance with  $\ell$  is the same. This is related to the fact that the behavior of  $\Gamma_2$  at low frequency is determined via the Kramers-Kronig transformation to the behavior of  $\Gamma_1$  also at high frequency. The latter depends strongly on the shape of the field. Only if this is assumed in order to reproduce the  $\omega^{-2}$ -behavior of the classical circuit, one can expect  $\gamma_1$  and  $\gamma_2$  to yield the same  $L_0$ .

In the capacitive region, where  $g \ll 1$ , the second terms on the right hand sides of (59) can be neglected. The first terms can be used to define an equivalent capacitance  $C$  and a dissipative resistance  $R$  by comparing with (58). For  $\alpha d \ll 1$ , in the limit of Coulomb interaction, we obtain from  $\gamma_2$

$$C \approx \frac{e^2}{h} \frac{4\beta_0}{\alpha v_F} \frac{1}{(1 - 2\beta_0 \ln \alpha d)^2}, \quad (60)$$

which is independent of the electric field and diverges for infinite interaction range,  $\alpha \rightarrow 0$ . By comparing with the first term of  $\gamma_1$  we obtain the dissipative resistance

$$R \approx \frac{h}{e^2} \frac{3}{32} \frac{1}{\beta_0} (1 - 2\beta_0 \ln \alpha d)^{3/2}. \quad (61)$$

For  $\alpha d > 1$ , we find

$$C \approx \frac{e^2}{h} \frac{d}{v_F} \frac{\beta_0}{(z^2 + \beta_0^2)^{3/2}} \exp \left[ \frac{\beta_0 + z^2}{8} \frac{\ell^2}{d^2} \right] \times \left[ 1 - \Phi \left( \frac{\ell}{2d} \sqrt{\frac{\beta_0 + z^2}{2}} \right) \right] \quad (62)$$

with  $z = \alpha d/2$  and  $\Phi$  the error function. Here, the capacitance depends on the electric field, for instance,

$$C = \frac{e^2}{h} C_1 \frac{g(1-g^2)}{v_F \alpha} \quad (63)$$

with  $C_1 = 2$  for  $g/\ell\alpha \gg 1$  and  $C_1 = (8\sqrt{2/\pi})(g/\ell\alpha)$  for  $g/\ell\alpha \ll 1$ . The corresponding dissipative resistances are

$$R = \frac{h}{e^2} R_1 \frac{g}{1-g^2} \quad (64)$$

with  $R_1 = 3/8$  and  $R_1 = (3\pi/256)(\alpha\ell/g)^2$  for  $g/\ell\alpha \gg 1$  and  $g/\ell\alpha \ll 1$ , respectively.

### C. Near resonance

As seen in fig. 4 the absorptive conductance shows a resonance near the frequency  $\omega_p$ . For this, the interaction has to be very strong, i.e.  $g \ll 1$ . Furthermore, the range of the electric field should not be too large (see below). Due to the scaling (cf. (22)) the results in the Coulomb and Luttinger limits are closely related. Indeed, in contrast to the limit  $\omega \rightarrow 0$ , in the Coulomb limit  $\alpha d \ll 1$  the conductance does not show any singular behavior near the resonance frequency  $\omega_m$ . Since near the resonance the zero-frequency resistance does not play any role,  $R_0$  and  $L_0$  are neglected when fitting  $C$ ,  $L$  and  $R$  to the ac-conductance. The parameters of the circuit can be obtained by fitting to the resonance frequency, and the width and the height of the resonance.

For small range of the electric field,  $\ell \ll q_p^{-1}$ , we find

$$C \propto \frac{e^2}{h v_F} \beta_1^{3/2} \lambda, \quad (65)$$

$$L \propto \frac{h}{v_F e^2} \beta_1^{1/2} \lambda, \quad (66)$$

$$R \propto \frac{h}{e^2} \beta_1^{1/2} \quad (67)$$

with  $\beta_1 = g$ ,  $\lambda = \alpha^{-1}$  and  $\beta_1 = g_0$ ,  $\lambda = d$  for  $\alpha d \gg 1$  and  $\alpha d \ll 1$ , respectively.

If, on the other hand, the range of the electric field is large,  $\ell > q_p^{-1}$ , the resonance becomes smaller and very broad. The parameters of the circuit depend here again on the field range. We find

$$C \propto \frac{e^2}{h v_F} \frac{\lambda^2}{\ell} \beta_1^2 \quad (68)$$

$$L \propto \frac{h}{v_F e^2} \ell \quad (69)$$

$$R \propto \frac{h}{e^2} \frac{1}{\beta_1} \frac{\ell^3}{\lambda^3}. \quad (70)$$

Remarkably, the capacitance obtained here scales in the same way with the system parameters as the one obtained in the limit of low frequency, cf. (63). Also the inductance scales as for  $\omega \rightarrow 0$ , although  $L \neq L_0$ . The dissipative resistance, however, scales differently and depends much stronger on  $\ell$  than  $L$  and  $C$ . This reflects, that the dissipative resistance is much more sensitive to the width and the height of the resonance than  $L$  and  $C$ . The product of the latter is fixed by the resonance frequency which is quite insensitive to the range of the electric field in a broad region of field ranges. If we assume  $L = L_0$  the scaling behavior of  $C$  near the resonance follows directly from  $\omega_m = (LC)^{-1/2}$ . Thus, the somewhat astonishing result of this is that one can identify a region of parameters for the 1D electron system, in which the scaling properties of  $L$  and  $C$  are independent of the frequency, though the numerical prefactors can be different.

### V. DISCUSSION

We considered the ac-transport properties of quantum wire with finite range interaction. The linear response theory was found to be exact, consistent with earlier work, as a result of the linearization of the dispersion relation. The dependence of the current on the electric field is given by the microscopic non-local conductivity. However, the latter is not very useful when aiming at a description of experiments. What is needed is a description in terms of externally accessible macroscopic quantities, as for instance given by Ohm's law. Such a relation can also be found in the present, non-local quantum region. By assuming the electric field to be non-zero near a given point  $x_j$  and the current to be detected by a probe at a point  $x_i$  one finds  $\Gamma(\omega) \equiv \Gamma_{ij}(\omega) = \sigma(x_i - x_j, \omega)$ . By generalizing to several probe positions  $x_1 \dots x_p$  one obtains

$$J_i(\omega) = \sum_j \Gamma_{ij}(\omega) U_j(\omega). \quad (71)$$

Such an approach has been used recently<sup>32</sup> in order to generalize the Landauer dc-approach to finite frequency. In the present work, the non-local conductances  $\Gamma_{ij}(\omega)$  are natural results of the response theory when suitable assumptions are made for the shape of the electric field. However, it seems to us that in the ac-regime this approach is not necessarily appropriate since it may be very difficult to apply experimentally ac-fields locally.

We find it more suitable to define a global average conductance via the time average of the absorbed power. If we consider a monochromatic electric driving field, this absorptive ac-conductance can be considered as the real part of a complex conductance. It provides information about the magnitude of the current in phase with the electric field. The imaginary part of the ac-conductance provides information about the phase shift between current and voltage. It was obtained by a Kramers-Kronig transformation from the real part.

We found that the absorptive ac-conductance factorizes into a product of the density of excitations and the Fourier-transformed auto-correlation function of the external electric field. A question related to this is whether the electric field to be used is the external one or whether one has to use the internal electric field that contains screening contributions. We found that one can use the external electric field in order to obtain the ac-conductance of the interacting system. If one uses the total field, the same result is obtained for the conductance. In any case, the result for  $\omega \neq 0$  depends on the spatial shape of the field.

Only in the dc-limit, the conductance becomes independent of the shape of the applied electric field and depends only on the applied external voltage. It is renormalized by the interaction parameter,  $\Gamma_1 = g e^2 / h$ . This does not contradict other recent results which indicate that for a Luttinger system of a finite length connected

to Fermi liquid leads the conductance is not renormalized by the interaction. We can argue that the interaction influences transport for system lengths  $L$  above  $q^{-1}(\omega)$ , the wave length of the charge density wave. When  $L < q^{-1}(\omega)$  the effect of the interaction can be neglected<sup>61,62</sup>. Since we are considering the thermodynamic limit, this region is outside the range of the validity of our model.

There is a resonance in the absorptive conductance at a frequency which corresponds to the inverse of a characteristic length scale of the interaction. In the limit of a 1D Coulomb interaction the latter is given by the cutoff length  $d$ . For the Luttinger limit, it is the interaction range  $\alpha^{-1}$ . Its height and width is influenced by the electric field, but its position is not. However, when having in mind a “realistic” situation, a word of caution is here in order. If one wants to use the Luttinger model as a model for a quantum wire, the parameters should be such that (i) linearization of the free-electron dispersion is a good approximation, i.e.  $q < k_F$ , and (ii) interband transitions are not important for the absorptive conductance,  $m^*\omega < \hbar d^{-2}$  ( $m^*$  effective mass). We have found above that  $q_p \propto d^{-1}$  in the Coulomb limit. The Fermi energy should be smaller than the interband energy distance. Therefore, the Fermi wave number is restricted to  $k_F < d^{-1}$ . Then,  $q_p \approx k_F$ , and corrections to linearization should be taken into account. Since furthermore  $\omega_m \propto v_F/d \propto \hbar/m^*d^2$  also interband transitions could become important near the resonance. For the Luttinger limit of the interaction the situation does not improve. One needs also to take into account interband mixing induced by the interaction for a proper description of the frequency region near the resonance.

However, at low frequency, and if the Fermi level is well below the onset of the second lowest subband, interaction-induced mixing of the bands can be neglected, and interband transitions are unimportant. Here, the real and the imaginary part of the conductance depend quadratically and linearly on  $\omega$ , respectively. The signs of the prefactors of these terms indicate capacitive- or inductive-like behavior of the system. By comparing the frequency behavior of the microscopic model with that of a “minimal” classical circuit of capacitances, inductances and resistances we find microscopic expressions for these quantities. They are, however, not unique. Their validity is restricted to certain parameter regions. Only the scaling properties are the same. Astonishingly, one can identify a parameter region, in which the scaling properties with the parameters of the system are the same for low frequency and near the resonance. This fact gives us some confidence that although the frequency region near the resonance is somewhat out of the range of validity of the model certain general features of the results remain valid.

In particular, the impedances depend in general strongly on the shape of the applied electric field. There is a competition between the range of the field and the range of the interaction, which determines whether the

system behaves as a capacitor ( $\ell\alpha \ll 1$ ) or as an inductor ( $\ell\alpha \gg 1$ ). In the former case, depending on the ratio between the interaction parameter  $g$  and  $\ell\alpha$ , the capacitance and the dissipative resistance may ( $g/\ell\alpha \ll 1$ ) or may not ( $g/\ell\alpha \gg 1$ ) depend on the field range. The infinite range of the interaction removes the dependence on the range of the field, such that one can interpret the impedances as genuine properties of the system that are only determined by the parameters of the microscopic model.

It is particularly interesting to note that the model allows to identify two conceptually different “resistances”. In the dc-limit, we have the equivalent of the “contact resistance”,  $\hbar/ge^2$ , which is however renormalized by the interaction parameter. Near the resonance frequency, one can define a “dissipative resistance”, approximately given by the inverse of the height of the resonance. The former depends on if and how the system is connected with the “outside world” via contacts. It changes to  $\hbar/e^2$  if the system length is smaller than the wave length of the charge density wave. In contrast, we expect that the latter is not changed as long as  $L > q_p^{-1} \propto d$ . By comparing with the classical circuit one also notes that the dissipative resistance is also present at low frequency, though it scales differently with the system parameters.

## VI. CONCLUSION

How to calculate the frequency dependent low-temperature electronic quantum transport properties of nanostructures when interaction effects are important is presently a subject of strong debate. Since experiment can access this regime by using modern fabrication technology, developing theoretical concepts for the treatment of transport processes taking into account interactions is important. In addition, future information technology will require high-frequency signal transportation in nano-scale systems. Proper theoretical understanding of the underlying microscopic processes will be essential for developing this technology in the far future.

In order to obtain insight into some of the peculiar microscopic features of this transport region, we have investigated the ac-response of a simple 1D model of an interacting electron system. We found that in spite of the simplicity of the model the definition of transport parameters provides already non-trivial questions to be solved. The transport parameters depend on the purpose for which they are supposed to be used. For instance, when asking for the current in some probe as a response to an electric field applied to another probe it is the non-local conductivity, which has to be used as the “conductance”. On the other hand, when being interested in the absorption of electromagnetic radiation, an average of the conductivity provides an “absorptive conductance”.

By starting from the “absorptive conductance”, we

have made an attempt to define for our system the quantum counterparts of the impedance parameters used in the transport theory of classical circuits. We found that they cannot be defined uniquely in terms of the parameters of the microscopic model, in accordance with the above general statement. However, certain scaling properties remain valid in astonishingly large regions of parameters. They even resemble the scaling properties of the corresponding classical definitions. This encourages to find similar quantities for more complicated systems as tunnel barriers and multiple tunnel barriers in the presence of interactions.

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