

FRIEDEL OSCILLATIONS IN LUTTINGER LIQUIDS

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Abstract. We study the density disturbance of a correlated one-dimensional electron liquid in the presence of a scatterer or a barrier. The $2k_F$ -periodic density profile away from the barrier (Friedel oscillation) is computed for arbitrary electron-electron interaction and arbitrary impurity strength. We find that in presence of correlations, the Friedel oscillation decays slower than predicted by Fermi liquid theory. In the case of a spinless Luttinger liquid characterized by an interaction constant $g \leq 1$, the asymptotic decay of the Friedel oscillation is x^{-g} . For a weak scatterer, the decay is even slower at small-to-intermediate distances from the impurity, with a crossover to the asymptotic x^{-g} law.

1. INTRODUCTION

One-dimensional (1D) interacting fermions have attracted a great deal of interest in the recent past spurred by the observation that the bosonization technique developed a long time ago [1]–[4] can be used to study the interplay between Coulomb interactions and disorder [5]–[12]. Theoretical investigations of quantum wires are also stimulated by the possibility of performing experiments probing such systems [13]. The last few years have seen a tremendous amount of activity, mostly devoted to transport quantities like the conductance [5]–[15]. In this work, we focus on the equilibrium electron *density* distribution of an interacting system with broken translational invariance [16]. In the presence of an impurity, conduction electrons will rearrange in order to screen the impurity charge. For dilute impurity concentration, it makes sense to first study the effect of a single impurity. Depending on the impurity strength, a barrier of arbitrary transmittance between zero and one hinders the electron flow. The corresponding

crossover between an insulator and a metal can significantly be affected by the Coulomb interaction between electrons. This issue touches upon important physical effects such as the pinning of a Wigner crystal [17] or of charge density waves [18], the breakdown of charging effects with increasing tunnel conductance [19], or quantum transport in 1D heterostructure channels [20, 21].

In the noninteracting case, the impurity is known to lead to a $2k_F$ -periodic decaying disturbance of the ground state density, the so-called Friedel oscillation [22]. In d dimensions, it has far away from the barrier the asymptotic form

$$\delta\rho(x) \sim \frac{\cos(2k_F x + \eta_F)}{x^d}, \quad (1)$$

where η_F is a phaseshift. In dimensions $d > 1$, the Coulomb interactions can presumably be incorporated by Fermi liquid theory such that the asymptotic x^{-d} behavior is still valid. Since Fermi liquid theory breaks down in 1D, the asymptotic decay law of the Friedel oscillation can now be modified by interactions. Indeed, as will be shown below, due to reduced screening, the asymptotic decay is always *slower* than the Fermi liquid $1/x$ law.

In order to properly describe Coulomb interactions, one has to specify the setup under consideration. If one deals with a 1D channel in gated heterostructures (quantum wire) [20, 21], the interactions are usually screened due to the presence of metallic gates near the channel. A short-ranged interaction leads to the Luttinger liquid model [4] described by a single dimensionless interaction constant g . The noninteracting Fermi liquid case corresponds to $g = 1$, and the presence of (repulsive) Coulomb interactions implies $g < 1$. On the other hand, one might as well consider a clean isolated channel where the $1/r$ tail of the Coulomb potential is not screened [23]. We will refer to this situation as the “long-ranged case” in the following. For the sake of clarity, we shall only consider a single transport channel, and most of our analysis will be concerned with the simplest case of spinless fermions. Electron–electron backscattering effects can then be incorporated by a renormalization of the interaction parameter g [24].

Qualitatively, Coulomb interactions favor a smooth density profile. Thus one expects that the Friedel oscillation should be smoothed and decay more slowly than predicted by Eq. (1). Furthermore, since the Coulomb interactions lead to a vanishing ground-state conductance in the presence of an impurity [5], one should expect a more efficient pinning of the Friedel oscillation, i.e., a larger amplitude than in the noninteracting case. Our results corroborate this simple picture. For the case of a spinless Luttinger liquid, the $1/x$ law is changed to an asymptotic x^{-g} decay far away from the barrier [16]. For a weak scatterer, the behavior of the Friedel oscilla-

tion is quite complicated. The decay is even slower than x^{-g} for small-to-intermediate distance from the barrier, with a crossover to the asymptotic x^{-g} law at some scale $x_0 \sim \lambda^{-1/(1-g)}$, where λ denotes a dimensionless impurity strength introduced below. These results have intrinsically many-body character and cannot be found from Hartree–Fock or related techniques.

In the presence of two impurities new features arise due to the possibility of resonant tunneling through the double barrier structure [25]. For the noninteracting case, it is easily shown that the Friedel oscillation on resonance decays faster than x^{-1} , namely with a x^{-2} law resulting from the interference of both contributions. Interactions will again modify the asymptotic properties of the on-resonance Friedel oscillation.

1.1. MODEL HAMILTONIAN

We treat the 1D interacting electron liquid in the framework of standard bosonization [1]–[4]. This approach is appropriate for low temperatures, where only excitations near the Fermi surface are relevant. In the following, we will mainly discuss the spinless case in detail. The creation operator $\psi^\dagger(x)$ for spinless fermions can equivalently be expressed in terms of the boson phase fields $\theta(x)$ and $\phi(x)$, which fulfill the algebra (we put $\hbar = 1$)

$$\begin{aligned} [\theta(x), \theta(x')]_- &= [\phi(x), \phi(x')]_- = 0 \\ [\phi(x), \theta(x')]_- &= -(i/2) \operatorname{sgn}(x - x'). \end{aligned} \quad (2)$$

Therefore $\Pi(x) = \partial_x \phi(x)$ is the canonically conjugate momentum to $\theta(x)$. With $\psi^\dagger(x) = \psi_+^\dagger(x) + \psi_-^\dagger(x)$, where the right- and left-moving parts are given by

$$\psi_\pm^\dagger(x) = \sqrt{\frac{\omega_c}{2\pi v_F}} \exp[\pm i k_F x + i\sqrt{\pi}(\phi(x) \pm \theta(x))], \quad (3)$$

the density operator $\rho(x) = \psi^\dagger(x)\psi(x)$ is readily found in the form

$$\rho(x) = \frac{k_F}{\pi} + \frac{1}{\sqrt{\pi}} \partial_x \theta(x) + \frac{k_F}{\pi} \cos[2k_F x + 2\sqrt{\pi}\theta(x)]. \quad (4)$$

This boson representation of the electron density operator is of essential importance for our work. The three terms are as follows. (1) The background charge density is k_F/π . (2) The density due to right- and left-movers results in the second term. (3) The last term originates from the mixed terms, i.e., from the interference between right- and left-movers. This $2k_F$ -term is responsible for the Friedel oscillation, since it has a nonzero expectation value if translational invariance is broken. The bandwidth cutoff ω_c is

defined as

$$\omega_c = v_F k_F , \quad (5)$$

which is equal to the Fermi energy for the Tomonaga dispersion relation.

The effective low-energy theory for a clean noninteracting fermion liquid is [4]

$$H_0 = \frac{v_F}{2} \int dx [\Pi^2 + (\partial_x \theta)^2] .$$

Neglecting electron-electron backward-scattering processes, the interaction among electrons is then described by the density-density interaction term

$$H_C = \frac{1}{2\pi} \int dx dx' \partial_x \theta(x) U(x-x') \partial_{x'} \theta(x') ,$$

where $U(x-x')$ is the (screened) Coulomb interaction potential. Most of our analysis is concerned with the Luttinger liquid case, where one has short-ranged interactions. We are then led to the Luttinger liquid Hamiltonian

$$H_L = \frac{v_F}{2} \int dx \left[\Pi^2 + \frac{1}{g^2} (\partial_x \theta)^2 \right] ,$$

where the interaction parameter

$$g = \frac{1}{\sqrt{1 + U_0/\pi v_F}} \leq 1$$

is related to the forward scattering amplitude U_0 , such that $g = 1$ represents the noninteracting case.

Let us now consider an elastic potential scatterer at $x = 0$ described by a potential $V(x)$. It leads to a contribution $H_I = \int dx V(x) \rho(x)$, and for a single δ -scatterer, $V(x) = \pi V k_F^{-1} \delta(x)$, one finds from Eq. (4) the generic form

$$H_I = \frac{\sqrt{\pi} V}{k_F} \partial_x \theta(0) + V \cos[2\sqrt{\pi} \theta(0)] . \quad (6)$$

The impurity strength V will often be given as dimensionless quantity

$$\lambda = \pi V / \omega_c . \quad (7)$$

Tuning λ from zero to infinity corresponds to changing the transmittance of the barrier from unity down to zero. The Hamiltonian is then $H = H_0 + H_C + H_I$, i.e.,

$$\begin{aligned} H &= \frac{v_F}{2} \int dx [\Pi^2 + (\partial_x \theta)^2] + \frac{1}{2\pi} \int dx dx' \partial_x \theta(x) U(x-x') \partial_{x'} \theta(x') \\ &+ \frac{\sqrt{\pi} V}{k_F} \partial_x \theta(0) + V \cos[2\sqrt{\pi} \theta(0)] . \end{aligned} \quad (8)$$

In the Luttinger liquid case the first two terms are replaced by H_L . The model (8) has been the subject of many studies in the past few years, primarily with regard to conductance computations [5]–[9].

1.2. GENERATING FUNCTIONAL

As is apparent from Eq. (4), one can compute $\langle \rho(x=y) \rangle$ from the generating functional $Z(y) = \langle \exp[2\sqrt{\pi} i\mu\theta(y)] \rangle$. We can first gauge away the forward-scattering term $\sim \partial_x\theta(0)$ in H_I by the unitary transformation $U = \exp[ig^2\lambda\phi(0)/\sqrt{\pi}]$. Then $Z(y)$ becomes

$$Z(y) = \langle \exp[2\sqrt{\pi} i\mu\theta(y)] \rangle e^{-i\mu g^2 \lambda \text{sgn}(y)}, \quad (9)$$

where the average has to be carried out using UHU^{-1} , which is just Eq. (8) without the forward-scattering term.

We formally solve for Z by introducing a field $q(\tau)$, which is constrained by

$$q(\tau) = 2\sqrt{\pi} \theta(x=0, \tau).$$

This constraint is enforced by a Lagrange multiplier field $\Lambda(\tau)$, such that one has the effective Euclidean action

$$\begin{aligned} S_e[\theta, \Lambda, q] &= \frac{v_F}{2} \int dx d\tau \left[\frac{1}{v_F^2} (\partial_\tau \theta)^2 + (\partial_x \theta)^2 \right] \\ &+ \frac{1}{2\pi} \int dx dx' d\tau \partial_x \theta(x, \tau) U(x-x') \partial_{x'} \theta(x', \tau) \\ &+ V \int d\tau \cos q(\tau) \\ &- 2\sqrt{\pi} i\mu \theta(y, 0) + i \int d\tau \Lambda(\tau) [2\sqrt{\pi} \theta(0, \tau) - q(\tau)]. \end{aligned} \quad (10)$$

The θ part of this effective action is Gaussian and can therefore be treated exactly by solving the classical Euler–Lagrange equation,

$$\frac{1}{v_F^2} \frac{\partial^2 \theta}{\partial \tau^2} + \frac{1}{g^2} \frac{\partial^2 \theta}{\partial x^2} = \frac{2\sqrt{\pi} i}{v_F} [\delta(x)\Lambda(\tau) - \mu\delta(x-y)\delta(\tau)]. \quad (11)$$

The solution of Eq. (11) is easily found in Fourier space,

$$\theta(x, \tau) = \int \frac{d\omega}{2\pi} \int \frac{dk}{2\pi} e^{i\omega\tau + ikx} \theta(k, \omega),$$

and similarly for $\Lambda(\tau)$. Then Eq. (11) takes the form

$$(\omega^2 + \omega_k^2) \theta(k, \omega) = -2\sqrt{\pi} i v_F (\Lambda(\omega) - \mu e^{-iky}),$$

where we have introduced the plasmon frequency

$$\omega_k = v_F |k| \sqrt{1 + U_k / \pi v_F} \quad (12)$$

with $U_k = \int dx \exp(-ikx) U(x)$. In the Luttinger liquid case, this is simply $v_s |k|$ with the sound velocity $v_s = v_F/g$. Defining the boson propagator functions

$$F(x, \omega) = v_F \int_{-\infty}^{\infty} dk \frac{\cos(kx)}{\omega^2 + \omega_k^2} \quad (13)$$

$$= \frac{\pi g}{|\omega|} e^{-|g\omega x|/v_F} \quad (\text{Luttinger liquid}), \quad (14)$$

one finds the solution

$$\theta(x, \tau) = \frac{-i}{\sqrt{\pi}} \int \frac{d\omega}{2\pi} e^{i\omega\tau} [\Lambda(\omega) F(x, \omega) - \mu F(x - y, \omega)]. \quad (15)$$

Next we have to determine the action corresponding to the classical solution. Inserting Eq. (15) into Eq. (10), we find after some algebra

$$\begin{aligned} S_{\text{cl}}[\Lambda, q] &= \int \frac{d\omega}{2\pi} \left\{ [\mu^2 + \Lambda(\omega)\Lambda(-\omega)] F(0, \omega) - 2\mu\Lambda(-\omega) F(y, \omega) \right. \\ &\quad \left. - i\Lambda(-\omega) q(\omega) \right\} + V \int d\tau \cos[q_0 + q(\tau)] - i\mu q_0, \end{aligned}$$

where q_0 is the zero-mode of the auxiliary field. Since S_{cl} is quadratic in Λ , the Lagrange multipliers are simply found by extremization. The result is

$$\Lambda(\omega) = \mu \frac{F(y, \omega)}{F(0, \omega)} + i \frac{q(\omega)}{2F(0, \omega)},$$

and inserting this into S_{cl} gives the generating functional (9) in the form of an average over the q field.

In the end, the generating functional takes the form

$$Z(x) = W(x) \mu^2 e^{-i\mu g^2 \lambda \text{sgn}(y)} \left\langle \exp \left[i\mu \left(q_0 + \int \frac{d\omega}{2\pi} q(\omega) \frac{F(x, \omega)}{F(0, \omega)} \right) \right] \right\rangle_q, \quad (16)$$

where the remaining q average has to be taken with the action

$$S[q] = \int \frac{d\omega}{2\pi} \frac{q(\omega)q(-\omega)}{4F(0, \omega)} + V \int d\tau \cos[q_0 + q(\tau)].$$

For a Luttinger liquid, we obtain therefore

$$S[q] = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{|\omega|}{4\pi g} |q(\omega)|^2 + V \int d\tau \cos[q_0 + q(\tau)]. \quad (17)$$

Apart from the kinetic energy term, this action corresponds to the action of a Brownian particle (q translates into the position of the particle) moving in a cosine potential under the influence of Ohmic dissipation [26]–[29]. The damping strength is connected with the Coulomb interaction constant g , and the height of the cosine potential is related to the impurity strength λ . The finite mass of the particle can be associated with the bandwidth ω_c .

The *envelope function* in Eq. (16)

$$W(x) = \exp \left[\int \frac{d\omega}{2\pi} \frac{F^2(x, \omega) - F^2(0, \omega)}{F(0, \omega)} \right] \quad (18)$$

does not depend on impurity properties at all and involves no q averaging. It will turn out that this function governs the asymptotic properties of the Friedel oscillation.

2. FRIEDEL OSCILLATIONS

In this section, we discuss the ground–state equilibrium Friedel oscillation induced by a barrier of arbitrary strength $\lambda = \pi V/\omega_c$ in the presence of Coulomb interactions. Most of our analysis will focus on the case of a spinless Luttinger liquid, but we mention some generalizations to the spin- $\frac{1}{2}$ situation or to long–ranged interactions. Since the density profile is symmetric around $x = 0$, we put $x \geq 0$ in the following.

2.1. GENERAL EXPRESSION

In the case of a spinless Luttinger liquid characterized by the interaction constant g , the boson propagator takes the form given in Eq. (14), namely

$$F(x, \omega) = \frac{\pi g}{|\omega|} \exp[-|\omega|x/v_s].$$

The envelope function (18) is therefore given by

$$W(x) = \exp \left[g \int_0^\infty d\omega \frac{e^{-2\omega x/v_s} - 1}{\omega} \right],$$

and we have to introduce an ultraviolet cutoff to regularize the integral. The appropriate cutoff ω_c is provided by the Fermi energy [see Eq. (5)], and employing an exponential cutoff function $\exp[-\omega/\omega_c]$, we obtain

$$W(x) = (1 + x/\alpha)^{-g},$$

with the microscopic lengthscale $\alpha = v_F/2g\omega_c$. Defining a “lattice spacing” $a = \pi v_F/\omega_c$, one has $\alpha = a/2\pi g$.

Let us now compute the deviation in the electron density profile, $\delta\rho(x) = \langle\rho(x)\rangle - k_F/\pi$, caused by the presence of the impurity. From Eq. (16), one can verify that away from the impurity the slow ($k \approx 0$) component in Eq. (4) is not affected by a potential scatterer. The only space-dependent density profile response to the impurity is the Friedel oscillation,

$$\delta\rho(x)/\rho_0 = -(1 + x/\alpha)^{-g} \cos(2k_F x - g^2\lambda) P(x) , \quad (19)$$

where $\rho_0 = k_F/\pi$. There is a renormalization of the noninteracting phase shift $\eta_F = \lambda$, which becomes $g^2\lambda$ in the case of a Luttinger liquid.

The *pinning function* $P(x)$ includes the nontrivial q average and takes the form

$$P(x) = - \left\langle \cos \left[q_0 + \int \frac{d\omega}{2\pi} e^{-g|\omega|x/v_F} q(\omega) \right] \right\rangle_q . \quad (20)$$

A useful quantity is the *pinning amplitude*

$$P_0 = -\delta\rho(x=0)/\rho_0 = P(0) ,$$

and it is also convenient to define the quantity

$$P_\infty = P(x \rightarrow \infty) = -\langle \cos q_0 \rangle_q .$$

From the definition of the pinning function, it is clear that $0 \leq P(x) \leq 1$ must be always fulfilled. Furthermore, the pinning function increases monotonically from P_0 to P_∞ .

For transmittance one ($\lambda = 0$), the ‘‘charge’’ q is free and $P(x) = 0$. For zero transmittance ($\lambda \rightarrow \infty$), the potential $V \cos q$ locks q at odd multiples of π , and P takes its maximal value, $P(x) = 1$, for all x . In that case, we obtain readily that the Friedel oscillation decays as x^{-g} . This result for zero transmittance can also be obtained by open boundary bosonization [11]. The slow algebraic decay of the Friedel oscillation cannot be reproduced by Hartree–Fock type calculations and is a true many–body effect. For instance, this can explicitly be seen by considering the limit $\lambda \rightarrow \infty$, where the Hartree–Fock type procedure devised by Matveev *et al.* [14] would predict a x^{-1} decay. In the following, we shall discuss the properties of the pinning function in some detail.

2.2. NONINTERACTING CASE: FERMIONIZATION

Let us first discuss the exact solution of the bosonized model (8) for the noninteracting case $g = 1$. For this particular value, we can obtain the exact solution for all quantities of interest by means of fermionization. This is seen by re–writing the Hamiltonian (8) for $g = 1$ in terms of the right– and left–moving fermion operators $\psi_\pm^\dagger(x)$ as given in Eq. (3). The impurity

term is a product of fermion operators at $x = 0$, and the bulk term becomes the massless Dirac Hamiltonian. In the end, H is equivalently expressed in the fermionized form

$$H^f = -iv_F \int dx \sum_{p=\pm} p \psi_p^\dagger(x) \frac{\partial}{\partial x} \psi_p(x) + v_F \lambda \sum_{p,p'=\pm} \psi_p^\dagger(0) \psi_{p'}(0) ,$$

where $\lambda = \pi V/\omega_c$ is the impurity strength (7).

Since H^f is quadratic in the fermion operators, it is sufficient to study the equations of motion

$$\left(\frac{1}{v_F} \frac{\partial}{\partial t} \mp \frac{\partial}{\partial x} \right) \psi_\pm^\dagger(x, t) = i\lambda \delta(x) (\psi_+^\dagger(x, t) + \psi_-^\dagger(x, t)) . \quad (21)$$

Away from $x = 0$, the solutions of Eq. (21) are simply free waves $\sim \exp(-iv_F k t \pm ikx)$, where $k > 0$ is the wavevector. At $x = 0$, the right- and left-moving components are discontinuous. Adding the two equations (21) and integrating over an infinitesimal region around $x = 0$, we obtain the jump condition

$$\left(\psi_+^\dagger - \psi_-^\dagger \right) (0^+) - \left(\psi_+^\dagger - \psi_-^\dagger \right) (0^-) = -2i\lambda \left(\psi_+^\dagger + \psi_-^\dagger \right) (0) ,$$

which implies that the ψ_\pm^\dagger are given in terms of the usual scattering waves [30] with k -independent transmission and reflection amplitudes, or equivalently a k -independent phase shift

$$\eta_k = \eta_F = \lambda . \quad (22)$$

In the absence of correlations, all effects of the impurity are contained in this phase shift, and the Friedel oscillation is readily evaluated in closed form. By expressing the reflection amplitude r_k in terms of the phase shift η_k , and then using the relation [30]

$$\delta\rho(x) = \frac{\text{Re}}{\pi} \int_0^{k_F} dk r_k e^{2ik|x|} ,$$

we obtain the ground-state result

$$\delta\rho(x) = \frac{\sin \eta_F}{2\pi|x|} [\cos(2k_F|x| + \eta_F) - \cos(\eta_F)] . \quad (23)$$

As expected for the noninteracting case, the 1D Friedel oscillation indeed decays as $1/x$.

2.3. SELF-CONSISTENT HARMONIC APPROXIMATION (SCHA)

Let us begin our discussion of the pinning function for $g < 1$ by describing a simple approximation based on Feynman's variational principle (self-consistent harmonic approximation, SCHA) [28, 31]. The most important approximation made in the SCHA is the neglect of tunneling transitions between different wells of the impurity cosine potential which seems reasonable for large λ . Therefore, we can assume q_0 to be an odd multiple of π and consider a Gaussian trial action

$$S_{\text{tr}}[q] = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{|\omega|}{4\pi g} |q(\omega)|^2 + \frac{\Omega}{2} \int d\tau q^2(\tau), \quad (24)$$

where the frequency Ω is determined from a variational principle for the free energy [32]. It states that the free energy F obeys the inequality

$$F \leq F_{\text{tr}} + \langle H - H_{\text{tr}} \rangle_{\text{tr}},$$

where the average has to be carried out using the trial action (24). Minimization of F yields

$$\Omega = V \exp \left[-\langle q^2 \rangle_{\text{tr}} / 2 \right].$$

From Eq. (24), we can read off

$$\langle q(\omega)q(-\omega') \rangle_{\text{tr}} = \frac{2\pi\delta(\omega' - \omega)}{\Omega + |\omega|/2\pi g}, \quad (25)$$

such that

$$\langle q^2 \rangle_{\text{tr}} = 2g \ln \left(1 + \frac{\omega_c}{2\pi g \Omega} \right),$$

and a self-consistency relation follows,

$$\Omega/V = \left(1 + \frac{\omega_c}{2\pi g \Omega} \right)^{-g} = \begin{cases} 1 & , \lambda \gg 1 \\ (2g\lambda)^{g/(1-g)} & , \lambda \ll 1. \end{cases} \quad (26)$$

For a strong scatterer $\lambda = \pi V/\omega_c \gg 1$, the trial frequency is simply V , as follows by a direct expansion of the impurity cosine term around the minima q_0 , which are odd multiples of π . In this limit, only small fluctuations about these minima are possible, with interwell transitions being forbidden by an exponentially small WKB tunneling factor. Therefore, we expect SCHA to be most valuable for the strong-scattering limit.

From Eq. (20) together with Eq. (25), one can easily evaluate the now Gaussian average. SCHA yields for the pinning function,

$$P(x) = \exp \left[-g e^{(x+\alpha)/x_0} E_1((x+\alpha)/x_0) \right], \quad (27)$$

with the exponential integral $E_1(y)$ [33] and the *crossover scale* x_0 given by

$$x_0/\alpha = \frac{1}{2g\lambda} (V/\Omega) = \begin{cases} 1/(2g\lambda) & , \quad \lambda \gg 1 \\ (2g\lambda)^{-1/(1-g)} & , \quad \lambda \ll 1, \end{cases} \quad (28)$$

where we have used Eq. (26) in the second step.

Since in the strong-scattering limit x_0 is even smaller than α , the term “crossover” is not meaningful in this limit. Using asymptotic properties of $E_1(y)$, Eq. (27) becomes for $x \gg \max(\alpha, x_0)$

$$P = e^{-gx_0/x} \simeq 1. \quad (29)$$

In the strong-scattering limit, the pinning function is essentially unity for all x , and the x^{-g} decay is always found. This result is in accordance with Monte Carlo results.

In the *weak-scattering limit*, $\lambda \ll 1$, the pinning function exhibits far more structure. The crossover scale goes to infinity as $\lambda \rightarrow 0$, namely $x_0 \sim \lambda^{-1/(1-g)}$. One expects that there are two different types of behavior for $x \ll x_0$ and $x \gg x_0$. Unfortunately, as we will see below, SCHA is unable to provide correct quantitative results except for very strong interactions, $g \ll 1$. That SCHA becomes more accurate for stronger interactions can be rationalized as follows. The presence of interactions leads, loosely speaking, to a renormalization of the barrier height. Using a perturbative renormalization group (RG) approach [5], one finds that λ grows under the RG transformation. For strong interactions, it flows quickly into the strong-scattering limit, where SCHA is essentially exact.

For $x \gg x_0$, SCHA always gives $P \simeq 1$ according to Eq. (29). This is an incorrect result, as can be seen from the exact result for the special case $g = 1$. That failure is due to the complete neglect of interwell tunneling in the SCHA. Without tunneling transitions, q_0 must be an odd multiple of π , and one finds $P_\infty = 1$ as predicted by SCHA. However, taking into account excursions to neighboring wells, it is readily seen that in general $P_\infty < 1$.

Despite of these shortcomings, the effective Gaussian treatment indicates that for a weak scatterer, there is a crossover, with a *slower* decay of the Friedel oscillation at small-to-intermediate distances $x \ll x_0$ than the asymptotic x^{-g} decay. In fact, Eq. (27) gives

$$P(x) = [(x + \alpha)/x_0]^g, \quad x \ll x_0,$$

which would imply that the Friedel oscillation does *not decay* at all up to the scale x_0 . Only for $x \gg x_0$, one would have the x^{-g} law.

In conclusion, for a weak scatterer, SCHA yields for the Friedel oscillation

$$\delta\rho(x)/\rho_0 = -\cos(2k_F x - g^2\lambda) \times \begin{cases} (x_0/\alpha)^{-g} & , \quad \alpha \ll x \ll x_0 \\ (x/\alpha)^{-g} & , \quad x \gg x_0. \end{cases} \quad (30)$$

This prediction is certainly incorrect for weak Coulomb interactions, see Eq. (23). The effective Gaussian treatment is only valuable for a strong scatterer or for strong Coulomb interactions. Furthermore, SCHA provides an estimate for the important crossover scale x_0 .

2.4. PERTURBATION SERIES FOR THE PINNING FUNCTION

To investigate the weak-scattering limit, we next attempt to evaluate the pinning function by perturbation theory in λ . The perturbation series is found by expanding the impurity propagator [27],

$$\exp \left[-V \int d\tau \cos q(\tau) \right] = \sum_{m=0}^{\infty} (-V/2)^m \int \mathcal{D}_m \tau \sum_{\{\sigma\}} \exp \left[i \sum_{j=1}^m \sigma_j q(\tau_j) \right],$$

where one has to sum over all auxiliary variables $\sigma_j = \pm 1$, and $\int \mathcal{D}_m \tau$ denotes a time-ordered integration over the m possible intermediate times τ_j . Thereby, the q average becomes Gaussian again, and $P(x)$ takes indeed the form of a power series in λ . One finds easily that only odd powers in λ contribute to the perturbational expansion.

We find for the lowest-order contribution

$$P(x) = \lambda \gamma_g^{(1)} (x/\alpha)^{1-g} + \mathcal{O}(\lambda^3), \quad (31)$$

with the numerical prefactor

$$\gamma_g^{(1)} = \frac{4^{g-1} \Gamma(g - \frac{1}{2})}{\sqrt{\pi} \Gamma(g)}, \quad (32)$$

where $\Gamma(z)$ is the Gamma function. This perturbative result is only valid for $g > \frac{1}{2}$, since otherwise $\gamma_g^{(1)}$ is not defined. That the computation of $P(x)$ is indeed a nonperturbative problem for all $g < 1$ can be seen by computing the higher-order terms in λ . From dimensional scaling, the perturbation series must have the form

$$P(x) = \sum_{m=1}^{\infty} \lambda^{2m-1} \gamma_g^{(m)} (x/\alpha)^{(2m-1)(1-g)}.$$

Since the higher-order terms increase faster, the first-order estimate (31) can only be valid for $x \ll x_0$. The crossover scale x_0 may be computed by equating the $m = 0$ and $m = 1$ components, or by arguing that $P(x) \leq 1$. Apart from numerical prefactors of order unity, these estimates for the crossover scale coincide with the SCHA crossover scale (28).

In contrast to Eq. (30), perturbation theory would therefore predict for the Friedel oscillation in the case of a weak scatterer $\lambda \ll 1$,

$$\delta\rho(x)/\rho_0 = -\cos(2k_F x - g^2\lambda) \times \begin{cases} \lambda\gamma_g^{(1)}(x/\alpha)^{1-2g} & , \quad \alpha \ll x \ll x_0 \\ P_\infty(x/\alpha)^{-g} & , \quad x \gg x_0 . \end{cases} \quad (33)$$

Here, we have assumed that the pinning function approaches a constant value $P_\infty \leq 1$ for $x \gg x_0$. This will be confirmed below by Monte Carlo simulations. The perturbative result (33) is expected to hold at least for weak interactions, $1 - g \ll 1$. Perturbation theory breaks down even for small distances, $x \ll x_0$, in the case of strong interactions, $g \leq \frac{1}{2}$.

In conclusion, perturbation theory predicts a similar weak-scattering scenario as SCHA. The Friedel oscillation exhibits a slower decay $\sim x^{1-2g}$ at small-to-intermediate distances from the barrier, with a crossover to the asymptotic x^{-g} law. The estimate for the crossover scale x_0 coincides in both approximations up to a numerical constant of order unity. From the scaling $x_0 \sim \lambda^{-1/(1-g)}$, one observes that the limit of a vanishing barrier implies a nontrivial long-distance behavior of the pinning function.

2.5. MONTE CARLO RESULTS

One can compute the pinning function (20) for any barrier height λ and arbitrary interaction constant g by employing numerically exact quantum Monte Carlo simulations. As our results for the strong-scattering limit corroborate the SCHA prediction $P(x) \simeq 1$, we only present numerical data for the weak-scattering case here. Numerical simulations are particularly useful for a determination of the behavior of the pinning function at small-to-intermediate distances from the impurity, $x \ll x_0$, where we expect to find a power law

$$P(x) \sim x^{\delta_g} . \quad (34)$$

The two approaches discussed above suggest a λ -independent exponent $\delta_g = g$ (SCHA) and $\delta_g = 1 - g$ (perturbation theory). The SCHA estimate should be valid for strong interactions, $g \ll 1$, while perturbation theory should hold at least for weak interactions, $1 - g \ll 1$.

To compute the pinning function by Monte Carlo, one has to consider finite temperatures, such that the frequency integrals become sums over Matsubara frequencies, $\omega_n = 2\pi n/\beta$, with $\beta = 1/k_B T$. We have checked that the temperatures used in our simulations were low enough to ensure that one is in the zero-temperature limit. Typically, $\beta\omega_c = 1000$ was sufficient. We have employed a hard cutoff scheme by keeping only the Matsubara frequencies with $|\omega_n| < \omega_c$. The Matsubara components $q(\omega_n)$ are then sampled according to the action (17) using the standard Metropolis algorithm. Our data were obtained on an IBM RISC 6000/Model 590

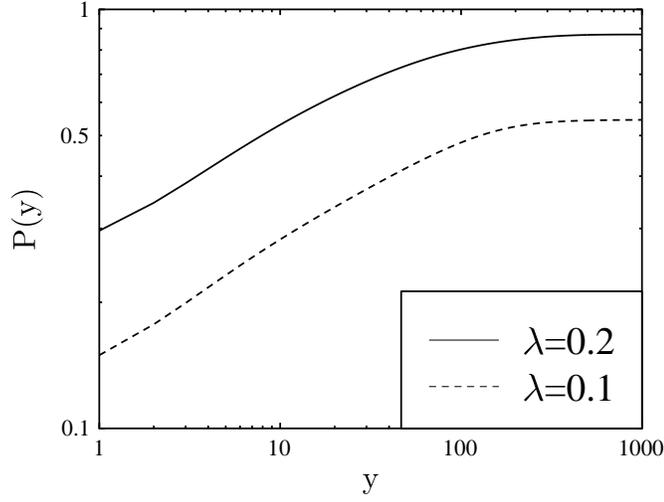


Figure 1. Monte Carlo data for the pinning function $P(y)$ at $g = \frac{1}{2}$ for two different values of the impurity strength. The dimensionless space variable is $y = \omega_c x / v_F$. Notice the logarithmic scales.

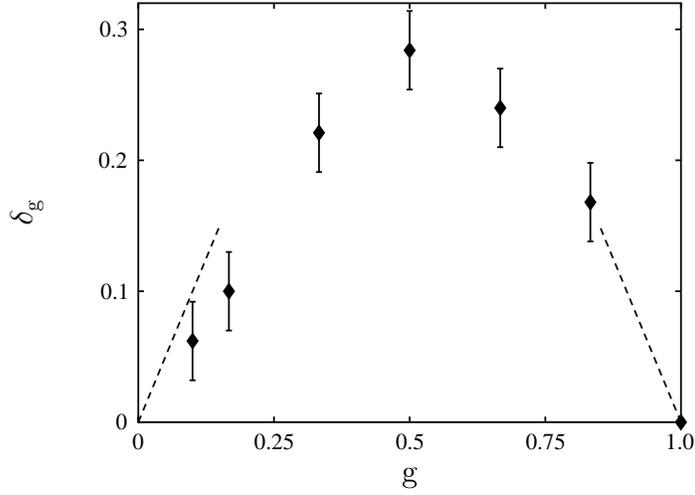


Figure 2. Numerical estimates for the exponent δ_g . Error bars were obtained from comparing data for different λ . The dashed curves are the SCHA estimate $\delta_g = g$ and the perturbative estimate $\delta_g = 1 - g$, respectively.

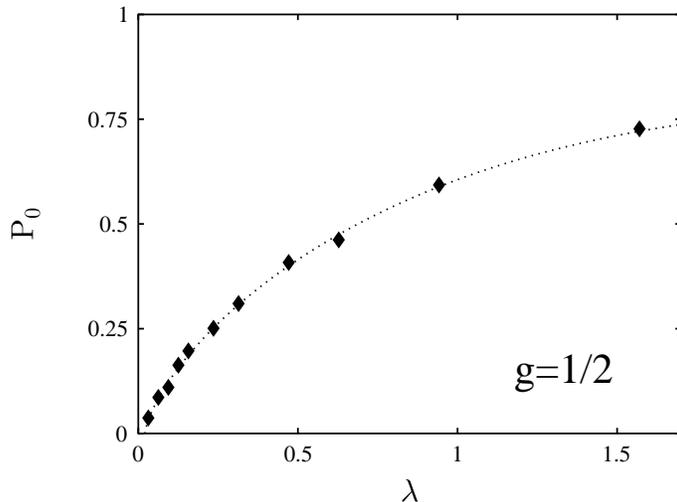


Figure 3. Monte Carlo data for the pinning amplitude P_0 at $g = \frac{1}{2}$ as a function of the impurity strength λ . The dotted curve is a guide to the eye only.

workstation using 50.000 samples for a given parameter set, with subsequent samples separated by 5 passes. One can compute the full pinning function for all x of interest in a single simulation run.

In Fig. 1, we show data for $g = \frac{1}{2}$ and the two impurity strengths $\lambda = 0.1$ and $\lambda = 0.2$. The data clearly display the crossover. We find the power law (34) for $x \ll x_0$, with an exponent $\delta_{1/2} = 0.28 \pm 0.03$. For $x \gg x_0$, the pinning function is essentially constant. The asymptotic decay of the Friedel oscillation at large $x \gg x_0$ is therefore x^{-g} , while the decay is slower for $x \ll x_0$,

$$\delta\rho(x) \sim x^{-g+\delta_g}.$$

As is apparent from Fig. 1, the region where this law is valid shrinks rapidly as the impurity strength is increased. Our data are generally consistent with the scaling $x_0 \sim \lambda^{-1/(1-g)}$.

Fig. 2 displays numerical values for δ_g . From our data, the power-law exponent δ_g appears to be independent of λ . The SCHA prediction $\delta_g = g$ is valid asymptotically for $g \rightarrow 0$, while for $1 - g \ll 1$, the perturbative result $\delta_g = 1 - g$ is reproduced, with a smooth turnover between both limits.

The ability of the scatterer to pin the charge density wave can conveniently be measured in terms of the pinning amplitude P_0 . As expected, P_0 increases with increasing impurity strength λ , see Fig. 3. To study the

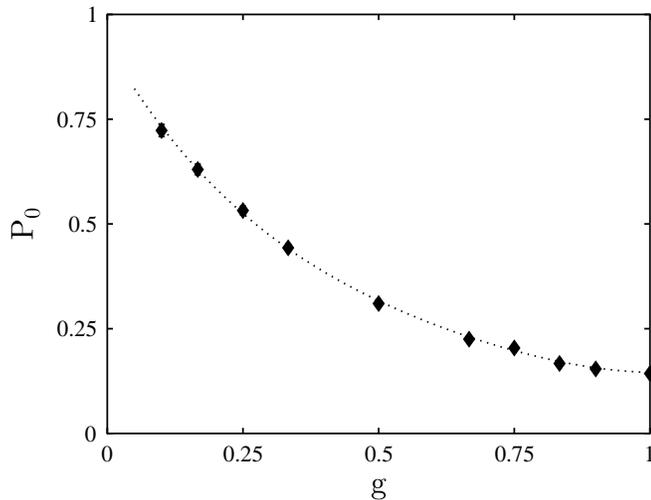


Figure 4. Monte Carlo data for the pinning amplitude as a function of the interaction constant g at fixed impurity strength $\lambda = \pi/10$. The dotted curve is a guide to the eye only.

dependence of P_0 on the interaction strength g , we show in Fig. 4 data for P_0 at fixed impurity strength. Clearly, the pinning is more efficient as the interaction becomes stronger. As mentioned in the introduction, this can qualitatively be understood in terms of the scaling of the barrier height [5]. Finally, in Fig. 5, we show the quantity P_∞ for $g = \frac{1}{2}$ as a function of λ . It shows the same qualitative behavior as in the noninteracting case. Most importantly, we generally find $P_\infty < 1$, in contrast to the SCHA prediction. Deviations from unity directly reflect tunneling transitions between different wells of the cosine potential.

One might wonder about computing the pinning function exactly at the special value $g = \frac{1}{2}$. Unfortunately, the fermionization method which is able to yield exact results for the conductance [26] cannot achieve this aim (except for the quantity P_0), since the density operator (4) cannot be expressed in terms of the fermions employed to diagonalize the $g = \frac{1}{2}$ model. In contrast, for vanishing interaction at the point $g = 1$, the fermionization method allows for an exact computation of the pinning function. To substantiate our results it would be of much interest to apply perturbed conformal field theory methods together with the thermodynamic Bethe ansatz to the interacting problem.

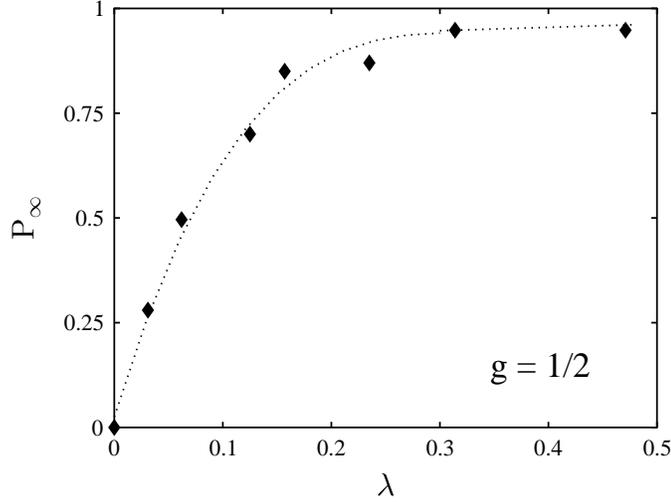


Figure 5. Monte Carlo data for P_∞ at $g = \frac{1}{2}$ as a function of the impurity strength. The dotted curve is a guide to the eye only.

2.6. LONG-RANGED INTERACTION

In the absence of metallic gates, one has to take into account the long-ranged character of the Coulomb interaction between the electrons in the 1D channel. The $1/x$ tail of the potential leads to a $k \rightarrow 0$ divergence of the Fourier transform,

$$\begin{aligned} U_k &= e^2 \int_{-\infty}^{\infty} dx \frac{\cos(kx)}{\kappa \sqrt{d^2 + x^2}} \\ &= (2e^2/\kappa) K_0(kd) \simeq (2e^2/\kappa) |\ln |kd||, \quad |kd| \ll 1, \end{aligned}$$

where d is the width of the 1D channel ($k_F d \ll 1$), and κ denotes the dielectric constant. The Bessel function $K_0(z)$ can be approximated by a logarithm here, and the corresponding logarithmic correction in the plasmon dispersion relation (12) is

$$\omega_k = \alpha_c v_F |k| \sqrt{|\ln |kd||},$$

where the dimensionless Coulomb interaction constant

$$\alpha_c = \sqrt{2e^2/\pi\kappa v_F}$$

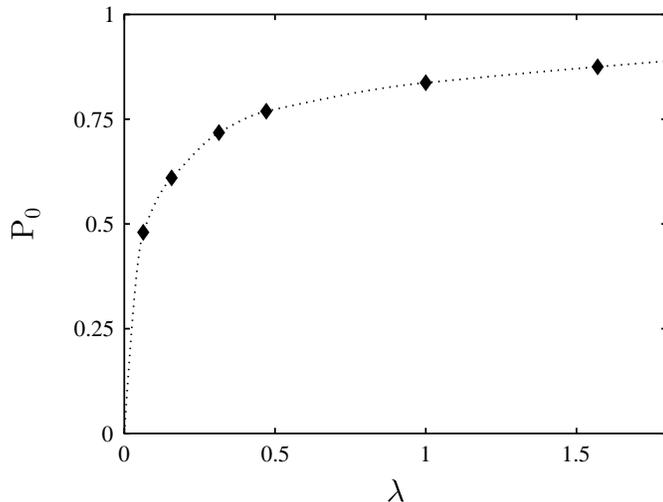


Figure 6. Monte Carlo data for P_0 as a function of impurity strength in the long-ranged case. We have taken $\alpha_c = 2.5$ and $d = \pi v_F / \omega_c$.

is of order unity in typical quantum wires [21].

Unfortunately, the boson propagator (13) cannot be evaluated in closed form anymore. However, one can obtain the asymptotic expansion for large x ,

$$F(x, \omega) = \frac{2v_F}{x\omega^2} \sin \left[\frac{|\omega|x}{\alpha_c v_F \sqrt{|\ln|\omega||}} \right]. \quad (35)$$

Remarkably, up to a prefactor of order unity, this expression holds even at $x = 0$, such that [9]

$$F(0, \omega) \sim \frac{1}{|\omega| \sqrt{|\ln|\omega||}}.$$

Computing the pinning function by Monte Carlo, we find that $P(x)$ is essentially constant, $P(x) \simeq P_0$, as could have been anticipated from the corresponding limit $g \rightarrow 0$ of the Luttinger liquid case. The functional dependence of P_0 on the impurity strength is depicted in Fig. 6.

For long-ranged interactions, the decay of the Friedel oscillation away from the barrier is completely governed by the envelope function $W(x)$ specified in Eq. (18). Asymptotic evaluation of the frequency integral using (35) shows that for $x \gg d$, the leading contribution is

$$W(x) \sim \exp \left[-\text{const.} \sqrt{\ln(x/d)} \right].$$

This decay is slower than any power law. In conclusion, in the presence of long-ranged unscreened Coulomb interactions, the Friedel oscillation takes the following form far away from the barrier

$$\delta\rho(x) \sim \cos(2k_F x) \exp\left[-\text{const.}\sqrt{\ln x}\right]. \quad (36)$$

This long-ranged density disturbance should lead to strong quasi-Bragg peaks in x-ray scattering.

2.7. SPIN- $\frac{1}{2}$ CASE

Let us now briefly comment on the spin- $\frac{1}{2}$ case. The bosonized treatment proceeds very similarly [16]. One can introduce charge fields $\theta_\rho(x), \phi_\rho(x)$ generalizing the fields $\theta(x)$ and $\phi(x)$ employed previously, and, in addition, spin fields $\theta_\sigma(x)$ and $\phi_\sigma(x)$. These fields are linear combinations of the respective spin-up and -down fields, such that the electron creation operator for spin $s = \pm$ at location x takes the form

$$\begin{aligned} \psi_s^\dagger(x) &= \sqrt{\frac{\omega_c}{2\pi v_F}} \sum_{p=\pm} \exp\left[ip\left(k_F x + \sqrt{\pi/2}[\theta_\rho(x) + s\theta_\sigma(x)]\right)\right] \\ &\times \exp\left[i\sqrt{\pi/2}[\phi_\rho(x) + s\phi_\sigma(x)]\right]. \end{aligned}$$

The various θ_ν fields ($\nu = \rho, \sigma$) commute among themselves, as do the ϕ_ν fields. The correct generalization of (2) is then given by

$$[\phi_\nu(x), \theta_{\nu'}(x')]_- = -(i/2)\delta_{\nu\nu'} \text{sgn}(x - x'),$$

and the canonical momentum for the θ_ν field is $\Pi_\nu = \partial_x \phi_\nu$.

We are concerned with density distributions in the presence of impurities or barriers. Generalizing Eq. (4), the bosonized form of the density operator for spin- $\frac{1}{2}$ electrons is

$$\begin{aligned} \rho(x) &= \rho_0 + \sqrt{2/\pi} \partial_x \theta_\rho(x) + \frac{2k_F}{\pi} \cos[2k_F x + \sqrt{2\pi} \theta_\rho(x)] \cos[\sqrt{2\pi} \theta_\sigma(x)] \\ &+ \text{const.} \cos[4k_F x + \sqrt{8\pi} \theta_\rho(x)], \end{aligned}$$

where the background charge is now $\rho_0 = 2k_F/\pi$. The three other terms are (1) the long-wavelength contribution, (2) the $2k_F$ charge density wave part, and (3) the $4k_F$ Wigner component [3, 23]. The Wigner component is not present in the spinless case, since two right-movers have to be flipped into left-movers simultaneously for this term to arise. Due to the Pauli principle, this can only happen for spin- $\frac{1}{2}$ electrons.

Assuming to be away from lattice or spin density wave instabilities, and neglecting electron–electron backscattering for the moment, the clean system is described by $H_0 = H_\sigma + H_\rho$ with

$$H_\rho = \frac{v_F}{2} \int dx \left[\Pi_\rho^2(x) + (\partial_x \theta_\rho(x))^2 \right] + \frac{1}{\pi} \int dx dx' \partial_x \theta_\rho(x) U(x-x') \partial_{x'} \theta_\rho(x').$$

The spin part H_σ is identical to the charge part H_ρ with no interaction potential U and the ρ fields replaced by the σ fields. Furthermore, the impurity backscattering contribution has the form

$$H_I = V \cos[\sqrt{2\pi} \theta_\rho(0)] \cos[\sqrt{2\pi} \theta_\sigma(0)].$$

Spin and charge parts are now coupled through this term. The forward-scattering contribution $\sim \partial_x \theta_\rho(0)$ leads only to a phase shift and is readily included by a gauge transformation.

Friedel oscillations can again be extracted from a generating functional generalizing Eq. (9),

$$Z(x, \mu_\nu) = \left\langle \exp \left[\sqrt{2\pi} i \sum_{\nu=\rho,\sigma} \mu_\nu \theta_\nu(x) \right] \right\rangle.$$

Since the impurity influences θ_ν only at $x = 0$, we constrain the field amplitudes $\theta_\nu(x = 0)$ to be equal to new fields, $q_\nu(\tau) = \sqrt{2\pi} \theta_\nu(0, \tau)$. We then proceed as before, and in the end, one is left with the nontrivial average over the q_ν fields alone, which are coupled to each other through H_I .

Collecting together all terms, we obtain

$$Z = -\mathcal{P}(x) \prod_{\nu=\rho,\sigma} W_\nu^{\mu_\nu^2/2}(x). \quad (37)$$

The envelope functions $W_\nu(x)$ generalizing (18) are

$$W_\nu(x) = \exp \left(\int \frac{d\omega}{2\pi} \frac{F^{(\nu)2}(x, \omega) - F^{(\nu)2}(0, \omega)}{F^{(\nu)}(0, \omega)} \right).$$

The charge and spin boson propagators are defined as in Eq. (13), and for a Luttinger liquid, we have again a single interaction constant $g = g_\rho \leq 1$. In the absence of a magnetic field and any spin-dependent interactions, one has to put $g_\sigma = 1$ to respect the SU(2) spin symmetry [4, 5]. Thus, the boson propagator functions read

$$F^{(\nu)}(x, \omega) = \frac{\pi g_\nu}{|\omega|} \exp[-|g_\nu \omega x|/v_F].$$

Finally, the quantity \mathcal{P} in Eq. (37) generalizes the pinning function. We find

$$\mathcal{P}(x) = - \left\langle \prod_{\nu=\rho,\sigma} \exp \left(i\mu_\nu \left[q_{\nu,0} + \int \frac{d\omega}{2\omega} q_\nu(\omega) \frac{F^{(\nu)}(x,\omega)}{F^{(\nu)}(0,\omega)} \right] \right) \right\rangle_q .$$

The q bracket stands for an average taking the action

$$S[q_\nu] = \sum_{\nu=\rho,\sigma} \int \frac{d\omega}{2\pi} \frac{|\omega|}{2\pi g_\nu} |q_\nu(\omega)|^2 + V \int d\tau \cos[q_{\sigma,0} + q_\sigma(\tau)] \cos[q_{\rho,0} + q_\rho(\tau)] .$$

In the following, we only discuss asymptotic properties far away from the barrier. For these, we can assume that \mathcal{P} is constant such that only the envelope functions $W_\nu(x)$ have to be evaluated. The spin part is like in the noninteracting case and gives rise to a $1/\sqrt{x}$ factor in the $2k_F$ Friedel oscillation. We note that there is the same crossover as discussed above, with a slower decay of the Friedel oscillation at $x \ll x_0$.

Combined with the charge channel, we find an asymptotic $\sim x^{-(1+g)/2}$ decay for the $2k_F$ Friedel oscillation. This is slightly faster than the corresponding x^{-g} law for spinless electrons and reflects that one starts to go away from the extreme 1D case by incorporating the second (spin) channel. As electron–electron backscattering cannot be treated by a simple renormalization of g in the spin- $\frac{1}{2}$ case, there could be weak logarithmic corrections to the $\sim x^{-(1+g)/2}$ law, depending on the magnitude of the backscattering amplitude [24]. Generalizing the two–channel spin–degenerate case discussed in this subsection to a multi–channel situation, one would then expect a x^{-1} law for large channel number, since Fermi liquid theory will eventually be valid in that case [7].

Remarkably, for spin- $\frac{1}{2}$ electrons, there is also a $4k_F$ Friedel oscillation component

$$\delta\rho(x) \sim \cos(4k_F x) x^{-2g} ,$$

which dominates over the $2k_F$ contribution for strong enough correlations, $g < \frac{1}{3}$. Since $4k_F$ corresponds to the inter–particle spacing, this suggests that for $g < \frac{1}{3}$ signatures of Wigner crystal behavior are induced by the impurity. The $4k_F$ component has been seen in numerically exact small–chain calculations by Hallberg and Balseiro [34]. In their Lanczos calculation, for a Hubbard chain with a magnetic impurity, the $4k_F$ component was found to be present for strong interactions.

Wigner crystal behavior has also been found by Schulz [23] for the clean system with long–ranged correlations. For $1/x$ interactions, the $4k_F$ Friedel oscillation decay is extremely slow. While the spin degrees of freedom involve again the $1/\sqrt{x}$ factor suppressing the $2k_F$ component, the $4k_F$ Friedel

oscillations decay again like $\exp(-c\sqrt{\ln x})$, i.e., slower than any power law. Effectively, one will then only observe the $4k_F$ component. In the spinless case, the same quasi long-ranged behavior appears for the $2k_F$ component already because the spin channel is absent, see Eq. (36).

2.8. FRIEDEL OSCILLATIONS NEAR A DOUBLE BARRIER

Let us now consider a double barrier arrangement which allows for the possibility of perfect transmission even in presence of Coulomb interactions. The resonant tunneling problem for a Luttinger liquid has been studied extensively before [25], and here we shall focus on the Friedel oscillation for a spinless single-channel Luttinger liquid. The symmetric double barrier structure considered is described by the Hamiltonian

$$H_I = V \sum_{\sigma=\pm} \cos[2\sqrt{\pi}\theta(\sigma R/2)] \quad (38)$$

instead of the single-barrier expression (6). We omit the forward-scattering contribution since it is inessential for the subsequent discussion. The resonance can then be tuned by varying $k_F R$ (or by varying a gate voltage coupling to the charge on the “island” formed between the barriers), and we employ again a dimensionless impurity strength $\lambda = \pi V/\omega_c$.

To study the Friedel oscillation for the interacting double-barrier problem, we proceed as in the single-barrier case and compute the generating functional (9). The θ field can be integrated out by introducing the two fields

$$q_{\pm} = 2\sqrt{\pi}\theta(\pm R/2),$$

where these constraints are enforced by Lagrange multiplier fields $\Lambda_{\pm}(\tau)$. The Gaussian integration over the θ degrees of freedom and the ensuing minimization of the Lagrange multiplier fields leave us with the following expression for the Friedel oscillation

$$\delta\rho(x)/\rho_0 = -W_2(x) \operatorname{Re} e^{2ik_F|x|} P_2(x)$$

with $\rho_0 = k_F/\pi$. The ground-state envelope function takes the general form

$$\begin{aligned} W_2(x) = & \exp \int \frac{d\omega}{2\pi} \left\{ -F(0, \omega) + \frac{F(0, \omega)}{F^2(0, \omega) - F^2(R, \omega)} \right. \\ & \times \left[F^2(|x + R/2|, \omega) + F^2(|x - R/2|, \omega) \right. \\ & \left. \left. - 2 \frac{F(R, \omega)}{F(0, \omega)} F(|x + R/2|, \omega) F(|x - R/2|, \omega) \right] \right\}, \end{aligned}$$

with the boson propagator $F(x, \omega)$ defined in Eq. (13). For the spinless Luttinger liquid, this gives to the right of the right barrier

$$W_2(x) = [1 + (x - R/2)/\alpha]^{-g}, \quad x > R/2,$$

where again $\alpha = v_s/2\omega_c$.

The pinning function $P_2(x)$ contains all dependency on impurity properties and can be expressed as average in $\{q_+, q_-\}$ space. It is more convenient to switch to the symmetric and antisymmetric combinations [25]

$$\begin{aligned} q &= (q_+ + q_-)/2 \\ Q &= (q_+ - q_-)/2 \end{aligned}$$

describing the transmitted charge q and the island charge Q , respectively. The action for the impurity averaging reads for finite temperatures

$$\begin{aligned} S[q, Q] &= \frac{1}{\beta} \sum_{n=-\infty}^{\infty} \frac{|\omega_n|}{2\pi g} \frac{q_n q_{-n}}{1 + \exp(-|\omega_n|R/v_s)} \\ &+ \frac{1}{\beta} \sum_{n=-\infty}^{\infty} \frac{|\omega_n|}{2\pi g} \frac{Q_n Q_{-n}}{1 - \exp(-|\omega_n|R/v_s)} \\ &+ 2V \int_0^\beta d\tau \cos[q(\tau)] \cos[k_F R + Q(\tau)], \end{aligned} \quad (39)$$

where q_n and Q_n are Matsubara components at frequency $\omega_n = 2\pi n/\beta$. The two modes are coupled by the impurity term. The Q mode has acquired a mass gap now [5].

For the spinless Luttinger liquid, the complex-valued pinning function $P_2(x)$ takes the general form

$$\begin{aligned} P_2(x) &= - \left\langle \exp \left[\frac{i}{\beta} \sum_n \left(\frac{e^{-|\omega_n(x+R/2)|/v_s} + e^{-|\omega_n(x-R/2)|/v_s}}{1 + e^{-|\omega_n|R/v_s}} q_n \right. \right. \right. \\ &+ \left. \left. \left. \frac{e^{-|\omega_n(x-R/2)|/v_s} - e^{-|\omega_n(x+R/2)|/v_s}}{1 - e^{-|\omega_n|R/v_s}} Q_n \right) \right] \right\rangle_{q, Q}. \end{aligned}$$

To the right of the right barrier, $x > R/2$, this yields the simpler expression

$$P_2(x) = - \left\langle \exp \left[\frac{1}{\beta} \sum_n e^{-|\omega_n|(x-R/2)/v_s} (Q_n + q_n) \right] \right\rangle_{q, Q}$$

where the average has to be taken using the action (39).

In the following, we shall focus on the case of small barriers. This has two reasons. (1) In the strong-scattering limit the resonances are very sharp,

and one might not be able to “find” the on-resonance Friedel oscillation. (2) A strong-scattering SCHA treatment, as applied to the single-barrier case previously, is not possible here since SCHA neglects tunneling and hence does not capture resonant tunneling. The SCHA prediction is always x^{-g} , which is only correct in the limit $\lambda \rightarrow \infty$ where the barriers reflect completely and no resonance is possible anymore.

In the weak-scattering case, we may compute the pinning function perturbatively in the impurity strength. To lowest order in λ , we obtain the Friedel oscillation

$$\delta\rho(x)/\rho_0 = -\lambda\gamma_g^{(1)} \sum_{\sigma=\pm} \cos\left(2k_F(x - \sigma R/2)\right) [(x - \sigma R/2)/\alpha]^{1-2g}, \quad (40)$$

with $\gamma_g^{(1)}$ defined in Eq. (32). This result is only valid for $g > \frac{1}{2}$ and $\lambda \ll 1$. Off resonance, we find the usual x^{1-2g} dependence, see Eq. (33). On resonance, the $2k_F$ component of the Fourier transform $V_k = 2V \cos(kR/2)$ of the scattering potential (38) vanishes [5]. Hence, the two cosine terms in Eq. (40) have different signs and interfere destructively. In that case, the Friedel oscillation has a different asymptotic behavior at $x \gg R$,

$$\delta\rho(x)/\rho_0 = -\lambda(2g - 1)\gamma_g^{(1)} \cos[2k_F(x - R/2)] \frac{R}{\alpha} \left(\frac{x}{\alpha}\right)^{-2g}. \quad (41)$$

The third-order contribution $\sim \lambda^3$ goes like x^{-4g} on resonance. Therefore, Eq. (41) holds in the asymptotic regime, contrary to the off-resonance case, where the higher order terms decay slower than the lowest-order contribution and thus lead to a modification of the perturbative prediction in the asymptotic regime (instead of x^{1-2g} , the correct asymptotic off-resonance law is x^{-g}).

In conclusion, for resonant tunneling through a double barrier, the Friedel oscillation can be dramatically affected if the barrier is tuned on resonance. The usual x^{-g} law is turned into the faster x^{-2g} law far away from the scatterers. This should be observable as a much weaker signal in x-ray scattering or NMR data.

3. CONCLUDING REMARKS

In this article, we have applied the bosonization method to compute Friedel oscillations for interacting fermions in one spatial dimension. The success of this method might come as a surprise, since density properties usually depend on details of the (noninteracting) electron band, such as band curvature. However, the asymptotic properties (which means several lattice spacings away from the barrier in that context) depend only on Fermi surface quantities and can be computed by bosonization.

We have shown that in 1D the Friedel oscillation in a correlated fermionic system decays slower than in the Fermi liquid case. This is in conflict with naive expectations based on the smeared momentum distribution of a Luttinger liquid. While in a Fermi liquid thermal smearing of the Fermi distribution induces an exponential decay of the Friedel oscillation, the smeared momentum distribution of a Luttinger liquid does not imply a faster decay. This shows again the failure of the quasiparticle picture for correlated fermions in 1D.

The results provided here have intrinsic many-body character and, to the best of our knowledge, cannot be obtained by other methods available at the moment. While results for the conductance can be obtained for weak interactions from a Hartree-Fock type approach [14], it is not possible to obtain the correct density profile with such methods. Furthermore, the fermionization technique allowing for exact results for the special interaction constant $g = \frac{1}{2}$ fails for the density profile as well. We hope our findings will motivate further theoretical work, e.g., on the effects of *magnetic* impurities on the screening cloud in one-dimensional interacting fermions.

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