Low-temperature transport of correlated electrons

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Transport properties of a single-channel Luttinger liquid impinging on a barrier have been studied for $g = 1/2 - \epsilon$, where g is the dimensionless interaction constant and $|\epsilon| \ll 1$. The relevant diagrams contributing to the conductance are identified and evaluated in all orders. Our approach represents a leading-log summation which is valid for sufficiently low temperature and small voltage. The asymptotic low-temperature corrections exhibit a turnover from the $T^{2/g-2}$ behavior to a universal T^2 law as the voltage is increased.

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The importance of many-body correlations for transport phenomena in quasi-one-dimensional (1D) quantum wires has attracted a lot of recent interest. Many theoretical studies treat the Coulomb interactions among the electrons in the Luttinger liquid framework, and then take into account one or a few impurities or barriers [1,2,3,4,5,6]. These studies are of direct relevance to the tunneling of edge state excitations in the fractional quantum Hall (FQH) regime [2,7], and to electron transport through a constriction in narrow high-mobility heterostructure channels [8]. Surprisingly few effort has been devoted to the *nonlinear* transport properties of such interacting 1D electron models. In this Letter, we provide rigorous results for the nonlinear dc conductance $G(V,T)$ in presence of strong interactions, namely near the value $g = 1/2$ of the interaction constant. Putting $g = 1/2 - \epsilon$ with $|\epsilon| \ll 1$, we compute the coefficients A_j in the series

$$G(V,T) = \sum_{j=0}^{\infty} A_j(V,T)\, \epsilon^j \ln^j[\max(k_{\rm B}T, eV)] \;, \qquad (1)$$

whereas contributions $\sim \epsilon^j \ln^k[\max(k_{\rm B}T, eV)]$ with $k < j$ are disregarded. Such a leading-log calculation is valid at sufficiently low temperature T and small voltage V,

$$|\epsilon| \ll 1 \;, \qquad eV/k_{\rm B}T_{\rm K} \ll 1 \;, \qquad T/T_{\rm K} \ll 1 \;, \qquad (2)$$

where $T_{\rm K}$ is the Kondo temperature.

If the Coulomb interaction between electrons does not contain unscreened long-range tails [9] and one is allowed to neglect backscattering, the 1D electron fluid behaves as a Luttinger liquid [10,11]. The interaction strength is then fully specified by a parameter g, where $g < 1$ for repulsive interactions and $g = 1$ for the noninteracting case. Introducing a pointlike scatterer to model the barrier, one finds several remarkable features [1,2,3]. Specifically, for $g < 1$ the conductance vanishes as $T \to 0$, implying that an arbitrarily weak barrier causes complete reflection. It is this nonperturbative regime we discuss here. Results in other regions of parameter space follow from duality relations [12] or by resorting to perturbation theory [1].

Low-temperature transport is of particular interest here. Scaling ideas have been employed by Kane and Fisher (KF) to piece together perturbative results in different parameter regimes. By matching onto the RG flows into the stable fixed point, they argued that the linear conductance in the nonperturbative regime should obey the power law $G(0,T) \sim T^{2/g-2}$ as $T \to 0$ [1]. On the other hand, there have been given arguments that the nonlinear conductance should exhibit a universal (g-independent) T^2 enhancement [13]. These results being correct would imply that the limits $V \to 0$ and $T \to 0$ do not interchange. The exactly solvable case $g = 1/2$ [1,13,14] cannot distinguish these two power laws and it is essential to study $g \neq 1/2$.

Very recently, the KF power law for the linear conductance appears to have been confirmed experimentally [7]. The tunneling of edge state excitations in the FQH regime for filling factor $\nu = 1/3$ is well described by the transport model considered here with $g = \nu$ [2], leading to a clean experimental setup undisturbed by backscattering. The findings of Ref. [7] have also received novel theoretical support. Integrability arguments can be exploited for $g = 1/3$, and using fermionization [5] and the thermodynamic Bethe ansatz [6], some exact results have been obtained. Our study is similar in spirit to recent calculations by Matveev, Xue and Glazman [3]. Their work assumes the presence of weak Coulomb interactions and therefore corresponds to a leading-log summation for $g = 1 - \epsilon$. Unfortunately, it is impossible to apply their method around smaller values of g which is important because the Kondo scale $T_{\rm K}$ vanishes with an essential singularity as $g \to 1^-$. In contrast, we perform a leading-log calculation for $g = 1/2 - \epsilon$. Finally, we mention that several quantum Monte Carlo results have been given in Refs. [15,16].

Within the standard bosonization procedure [1,11], the fermionic field operator is described in terms of two bosonic fields $\phi(x)$ and $\theta(x)$ satisfying the equal-time

commutator $[\phi(x),\theta(x')] = (-i/2)\mathrm{sgn}(x-x')$. Omitting irrelevant terms, the Hamiltonian density describing the low-energy behavior is [1]

$$\mathcal{H}(x) = \frac{g}{2}(\nabla\phi)^2 + \frac{1}{2g}(\nabla\theta)^2 + V_0\,\delta(x)\cos[2\sqrt{\pi}\theta] + eV\,\delta(x)\,\theta/\sqrt{\pi}\;. \qquad (3)$$

The (weak) barrier is modelled by a short-range scattering potential centered at $x = 0$, and the coupling constant V_0 is proportional to its Fourier transform at $2k_\mathrm{F}$. The last term represents an external static voltage V.

There exists a profound connection between this Luttinger liquid picture and the dissipative quantum diffusion of a light particle in a periodic potential. We will exploit an exact formal mapping between the Luttinger model (3) and the dissipative tight-binding (TB) model discussed in Refs. [12,13,14]. The central parameters in the dissipative TB model are the hopping matrix element Δ, the dimensionless Ohmic system-bath coupling K and the bias σ. To avoid spurious divergences, one also needs to introduce a cutoff frequency ω_c for the bath spectral density, which amounts to the bandwidth in the Luttinger model [10]. In effect, there is a one-to-one correspondence between the Ohmic bath modes and the plasmon modes of the Luttinger liquid away from the impurity. The current-voltage characteristics for transport of a Luttinger liquid is thereby expressed in terms of the mobility-bias relation for the dissipative TB model.

This formal equivalence can be shown directly via unitarily transforming the Hamiltonian (3) onto the TB Hamiltonian. Employing the standard definition of the nonlinear mobility $\mu(\sigma,T)$ [12,13] and denoting the free Brownian mobility by μ_0, we find for the current-voltage characteristics

$$I(V,T)/G_0V =: G(V,T)/G_0 = 1 - \mu(K\sigma,T)/\mu_0 \qquad (4)$$

with $G_0 = ge^2/h$. The necessary parameter identifications are

$$g = K\;, \qquad V_0 = \hbar\Delta\;, \qquad eV = \hbar\sigma\;. \qquad (5)$$

In the remainder, we discuss the nonlinear mobility for $K = 1/2 - \epsilon$ (where $|\epsilon| \ll 1$), and then draw conclusions concerning the original transport problem from Eq.(4).

To compute the mobility, we employ Feynman-Vernon theory. One can envision the possible paths of the TB particle as moving on a lattice spanned by the forward (q) and backward (q') real-time paths, with the action containing a nonlocal influence functional due to the eliminated bath modes. Switching to symmetric and antisymmetric combinations of q and q', the symmetric ones can be integrated out, and it suffices to sum over the off-diagonal "charges" $\xi_j = \pm 1$ indicating the hopping direction of the antisymmetric path. In the end, the mobility takes the form of a power series in Δ^2 [12,13],

$$\mu(\sigma,T)/\mu_0 = (2\pi K/\sigma)\,\mathrm{Im}\,U(\sigma,T)$$

with U describing an interacting kink gas

$$U = \sum_{m=1}^{\infty}(-1)^m\Delta^{2m}\int_0^\infty d\tau_1\cdots d\tau_{2m-1}\sum_{\{\xi\}} \times \exp\Big[-i\sigma\sum_j p_{j,m}\tau_j + \sum_{j>k}\xi_j S(\tau_{jk})\xi_k\Big] \times \prod_{j=1}^{2m-1}\sin(\pi p_{j,m}K)\;, \qquad (6)$$

where the charges $\xi_j = \pm 1$ have to obey overall neutrality, $\sum_j \xi_j = 0$. The interaction potential is

$$S(\tau) = 2K\ln[(\hbar\omega_\mathrm{c}/\pi k_\mathrm{B}T)\sinh(\pi k_\mathrm{B}T\tau/\hbar)]\;.$$

We use the quantities $p_{j,m} = \sum_{i>j}\xi_i$ to measure how far off-diagonal a given path consisting of $2m$ charges is. With the convention $\xi_1 = -1$, all $p_{j,m}$ have to be positive integers. Finally, the time interval between the two hops ξ_j and ξ_k is denoted as τ_{jk}.

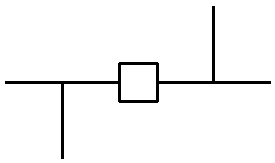

FIG. 1. Diagram resulting in the contribution $U^{(1)}$ to the mobility. Time flows along the horizontal line and vertical lines symbolize charges $\xi = \pm 1$.

For $K = 1/2 - \epsilon$, the phase factors appearing in Eq.(6) are

$$\sin(\pi pK) \simeq \begin{cases} (-1)^{(p-1)/2} & ,\quad p \text{ odd}, \\ \pi\epsilon p\,(-1)^{(p-2)/2} & ,\quad p \text{ even}. \end{cases} \qquad (7)$$

If we consider, e.g., the charge configuration $\xi = (--++)$, the contribution to U will have a $2\pi\epsilon$ prefactor. On the other hand, the breathing mode integral of the interior dipole has a $1/\epsilon$ singularity arising from the $1/\tau^{1-2\epsilon}$ short-time behavior of the intradipole interaction factor $\exp[-S(\tau)]$. Together with the phase prefactor, the dipole gives the finite breathing mode contribution $\bar\gamma$ with $\bar\gamma = \gamma(\omega_\mathrm{c}/\gamma)^{2\epsilon}$, where $\gamma \equiv \pi\Delta^2/\omega_\mathrm{c}$. The frequency scale $\bar\gamma$ defines the Kondo temperature T_K (see below). It is then convenient to split up the intradipole time interval into a short-time part $0 < \bar\gamma\tau < 1$ ("collapsed dipole") and a remaining long-time part.

Next we observe that a collapsed dipole has *no* interactions with other charges. Therefore, the grand canonical sum over all possible arrangements of collapsed dipoles between two confining charges is done easily. Generally, there are two types of collapsed dipoles, namely $(-+)$

and $(+-)$. Because of the phase factors (7), these cancel each other completely if the exterior charges define an even p value (one has to make an excursion to odd p which leads to a factor ± 1 depending on dipole type). However, if one has odd p between the outer charges, the respective phase factors for the two types of collapsed dipoles are different in magnitude. As a consequence, one has to insert a dilute gas of them. With the two exterior charges spanning a time interval τ, this insertion simply results in a factor $\exp(-\bar{\gamma}\tau)$. We shall mark this factor by a square in the diagrams below.

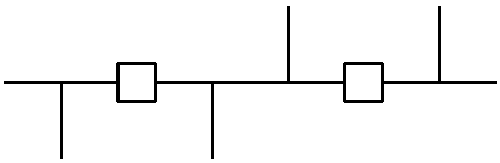

FIG. 2. Diagram giving the contribution $U^{(2)}$.

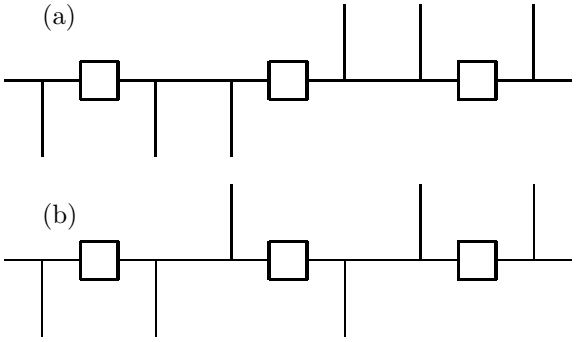

FIG. 3. Diagrams resulting in the contribution $U^{(3)}$.

To proceed further, we split up U into terms $U^{(n)}$ each of which contributes to order ϵ^{n-1} (and higher orders). The quantity $U^{(1)}$ corresponding to the diagram in Fig. 1 gives already the exact solution for $K = 1/2$ [13]. Generally, the quantity $U^{(n)}$ is the sum of all diagrams with $2n$ charges of total charge zero which cannot be divided into neutral clusters. These diagrams are dressed by inserting a gas of collapsed dipoles at every odd time interval. There are no insertions of collapsed dipoles at even time intervals for the reason given above, and these time integrations are restricted to $\bar{\gamma}\tau > 1$ since collapsed contributions are already included in lower-order diagrams. The diagram leading to $U^{(2)}$ is shown in Fig. 2, which together with $U^{(1)}$ yields all terms in order ϵ. To give a final example, one has to evaluate the two diagrams shown in Fig. 3 if one is interested in $U^{(3)}$.

We have extracted the leading logarithms by applying the following systematic procedure. (1) Draw all possible diagrams contributing to $U^{(n)}$. (2) Consider the charge interactions $S(\tau)$ for $\epsilon = 0$ first. (3) Apply a decomposition theorem to reduce the interactions to pair interactions. In diagrammatic parlance, this can be achieved by grouping the $2n$ charges into n pairs where paired charges must have opposite sign. By virtue of such a Wick theorem, one obtains $n!$ graphs per diagram. The sign of each graph is determined by the number of crossings if one connects paired charges. (4) Group the graphs into classes regarding the dependence on even time intervals. These are the relevant time integrations which turn out to be responsible for leading-log contributions. (5) Evaluate every class in leading-log accuracy [in the sense of Eq.(1)]. The remaining odd time integrations can be reduced to standard integrals. (6) Take into account the ϵ-dependence of the interactions again.

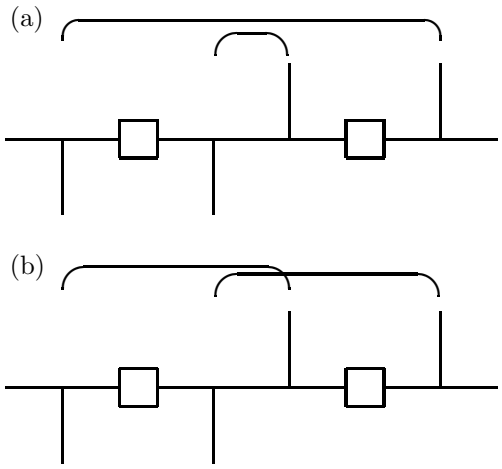

FIG. 4. The 2! graphs obtained from the decomposition theorem for the contribution $U^{(2)}$ shown in Fig. 2. The relative signs are + for graph (a) and − for graph (b). The curves display the interactions between paired charges.

To illustrate these rules, the diagram shown in Fig. 2 can be decomposed into the two graphs depicted in Fig. 4. It is then straightforward to find all leading-log contributions due to these two graphs. Similarly, higher-order diagrams can be evaluated in leading-log approximation. Computational details will be given elsewhere [17].

It is still a formidable task to evaluate all diagrams contributing to the coefficients $A_j(\sigma, T)$ in Eq.(1). Fortunately, for the asymptotic low-energy properties, it is sufficient to calculate them to lowest order in σ and T. Since $A_j(\sigma, T)$ can be expanded in powers of σ^2 and T^2, the leading corrections come from $A_j \sim \sigma^2, T^2$. The diagrams contributing to these terms can be classified and evaluated *in all orders in* ϵ, yielding the exact asymptotic behavior under conditions (2). The relevant diagrams in order n consist of external charge pairs $(--)$ and $(++)$, with $n-2$ dipoles of finite length inserted in between (see Fig. 3 for $n = 3$). One has again two types of extended dipoles and therefore 2^{n-2} diagrams in order n. We have evaluated the leading logs due to these diagrams for any n. All remaining diagrams do not contribute to the asymptotic low-energy behavior.

For the leading low-energy corrections, we find the exact result

$$\mu(\sigma,T)/\mu_0 = 1 - (1/3)\left[(\sigma/\bar{\gamma})^2 + (\pi k_{\rm B}T/\hbar\bar{\gamma})^2\right]^{1+4\epsilon} \pm \cdots \tag{8}$$

If the external bias exceeds the thermal scale, $\hbar\sigma \gg k_{\rm B}T$, Eq.(8) gives ϵ-independent low-temperature corrections $\sim T^2$ to the nonlinear zero-temperature mobility. On the other hand, for $\hbar\sigma \ll k_{\rm B}T$, the asymptotic low-temperature corrections are $\sim T^{2+8\epsilon}$. Thus our dynamical calculation corroborates the KF scaling law. Eq.(4) with (8) describes a smooth turnover from the KF law $G \sim T^{2/g-2}$ to the universal T^2 low-temperature behavior as the voltage is increased. For any nonzero voltage, the ultimate low-temperature behavior will be T^2.

These findings demonstrate that the limits $V \to 0$ and $T \to 0$ do not commute. The anomalous exponent $1+4\epsilon = 1/g - 1$ in (8) reflects the critical nature of the model at $T = V = 0$. The turnover from the critical $T^{2/g-2}$ law to the analytical T^2 enhancement is due to the breakdown of scale invariance when an external voltage is applied [18]. The physical origin of the T^2 law is the low-frequency thermal noise associated with the Ohmic nature of the plasmon modes in the leads. Eq.(8) describes the full turnover quantitatively. We wish to stress that backscattering (BS) cannot alter the T^2 behavior. In the spinless case, BS is treated as exchange event of forward scattering. As this simply leads to a redefinition of g, the above results remain valid. In the spin-$\frac{1}{2}$ case, BS gives weak logarithmic temperature corrections to the $T^{2/g-2}$ law at zero voltage [3]. For finite voltage, the RG analysis shows that these corrections are not present, and one finds again the T^2 law.

The KF law for the linear conductance holds for temperatures

$$T \ll T_{\rm K} \equiv (V_0/\hbar\omega_{\rm c})^{g/(1-g)}\, \pi V_0/k_{\rm B}\ .$$

This estimate for the Kondo temperature follows from the convergence radius of the series (6), and for $g = 1/2 - \epsilon$, the quantity $\bar{\gamma}$ emerges indeed. For $T \gg T_{\rm K}$, one can safely apply perturbation theory. The $T^{2/g-2}$ scaling has been observed in the above-mentioned $g = 1/3$ FQH transport experiments [7]. As the source-drain voltage drop was of the order $eV/k_{\rm B}T \approx 0.2$, the voltage was unable to spoil the KF T^4 behavior. It would be interesting to perform these experiments at voltage $eV \gg k_{\rm B}T$, where the predicted T^2 enhancement should be observable. Sample heating is not expected to cause serious problems since the conductance remains small.

Finally, we briefly address the higher powers of σ^2 and T^2 in the coefficients A_j. For the next-leading contributions $A_j \sim \sigma^4, T^4, \sigma^2T^2$, diagrams of the type $(--X--X++X++)$, where X stands for a grand canonical gas of finite-length dipoles, have to be considered in addition to the diagrams $(--X++)$ leading to Eq.(8). Our results show, e.g., that the next-leading corrections in Eq.(8) behave as $T^{4+16\epsilon}$ (for $\sigma = 0$). In addition, the full asymptotic expansion of the low-temperature nonlinear conductance can be deduced in closed form [17]. We believe that our leading-log technique will also be valuable for exact noise calculations in the asymptotic regime.

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